

Name _____

Date _____

Unit 9 · Sequences and functions — revision sheet

Everything you need from every section. Read it before the end-of-unit test.

9.1 Generating sequences

- A **sequence** is an ordered list; each number in it is a **term**.
- A **term-to-term rule** says how to get from one term to the next.
- To generate a sequence you need the rule **and** the first term — one without the other is not enough.
- A sequence is **linear** when the gap between terms is always the same. That gap is the **common difference**.
- To find the common difference, subtract any term from the one after it — and check the gap really is constant.
- To reach the k th term from the first, take $k - 1$ steps, not k . That off-by-one is the commonest slip.
- Not every sequence is linear: 2, 6, 18, 54 multiplies by 3; 1, 4, 9, 16 squares the position.
- Learn the square, cube and triangular numbers by sight — they come up all year.

Key words: sequence · term · term-to-term rule · first term · common difference · linear sequence · square numbers · triangular numbers

3, 7, 11, 15, 19, ...
 1st term 3 2nd term 7 3rd term 11
 first term 3, rule "add 4"
 $3 \rightarrow 7 \rightarrow 11 \rightarrow 15 \rightarrow 19$
 rule "add 4", first term 1 $\rightarrow$ 1, 5, 9, 13
 rule "add 4", first term 3 $\rightarrow$ 3, 7, 11, 15
 5, 9, 13, 17

9.2 Using the n th term

- The **n th term** rule gives any term straight from its **position**, with no stepping.
- $n = 1$ for the first term, $n = 2$ for the second, and so on.
- The **coefficient of n is the common difference**. A gap of 4 means the rule starts $4n$.
- Find the constant by comparing $4n$ with the sequence: what must be added to reach it?
- For a decreasing sequence the coefficient is negative: 20, 17, 14 has rule $-3n + 23$.
- **Always check** by substituting $n = 1, 2, 3$. It takes seconds and catches almost everything.
- To find a term's position, set the rule equal to it and solve.
- If solving gives a fraction, that number is **not** in the sequence — positions must be whole numbers.

Key words: n th term · position · coefficient · substitute · position-to-term rule

term-to-term: $3 \rightarrow 7 \rightarrow 11 \rightarrow 15 \rightarrow \dots$ 99 steps
 n th term $4n - 1$: put $n = 100 \rightarrow 399$
 5, 9, 13, 17 gap 4 $\rightarrow$ starts $4n$
 20, 17, 14 gap $-3 \rightarrow$ starts $-3n$
 want 7, 10, 13, 16
 $3n$ gives 3, 6, 9, 12
 always 4 more $\rightarrow 3n + 4$

9.3 Functions and function machines

- A **function** turns each input into exactly one output.
- A **function machine** shows the operations in order, left to right.
- To go forwards, apply the operations in order.
- To go **backwards**, use the **inverse** of each operation, in **reverse order**.
- Inverses: $+$ and $-$; $\times$ and $\div$.
- Order matters. " $\times 2$ then $+ 1$ " is $y = 2x + 1$; " $+ 1$ then $\times 2$ " is $y = 2(x + 1)$.
- A **mapping diagram** draws an arrow from each input to its output.
- Writing the machine as $y = \dots$ turns it into algebra, which is where Unit 11 picks it up.

Key words: function · function machine · input · output · inverse · mapping diagram

input 4 $\rightarrow$ $[\times 3] \rightarrow$ output 12
 input 7 $\rightarrow$ $[\times 3] \rightarrow$ output 21
 input 5 $\rightarrow$ $[\times 2] \rightarrow 10 \rightarrow [+ 1] \rightarrow 11$
 input 5 $\rightarrow$ $[+ 1] \rightarrow 6 \rightarrow [\times 2] \rightarrow 12$
 different order, different answer
 output 25, machine $\times 3$ then $+ 4$
 undo $+ 4$: $25 - 4 = 21$