Name Date

Unit 8 · Shapes and symmetry — teaching slides

24 slides, formatted for printing.

8.1 Polygons

What counts as a polygon

- A closed shape made only of straight sides.
- It has as many angles as it has sides.
- A circle is not a polygon — no straight sides.
- The name comes from the number of sides.

```
3 sides triangle 6 sides hexagon
4 sides quadrilateral 8 sides octagon
5 sides pentagon 10 sides decagon
```

Regular means both

- **Regular** needs equal sides **and** equal angles.
- A rhombus has equal sides but unequal angles — not regular.
- A rectangle has equal angles but unequal sides — not regular.
- A square has both, so a square is a regular quadrilateral.

```
rhombus sides ✓ angles ✗
rectangle sides ✗ angles ✓
square sides ✓ angles ✓ → regular
```

Where the angle sum comes from

- Draw every diagonal from one vertex.
- An n-sided polygon splits into $n - 2$ triangles.
- Each triangle contributes 180°.
- So the angles total $180(n - 2)°$ — you can derive it, not memorise it.

```
quadrilateral: 2 triangles → 360°
pentagon: 3 triangles → 540°
hexagon: 4 triangles → 720°
```

Exterior angles always total 360°

- Imagine walking right round the outside of the shape.
- At each corner you turn through the exterior angle.
- By the time you are back where you started you have turned once.
- One full turn — 360° — whatever the shape.

```
regular pentagon: 360 ÷ 5 = 72° each
regular octagon: 360 ÷ 8 = 45° each
regular decagon: 360 ÷ 10 = 36° each
```

The quick route

- Interior and exterior angles at a vertex add to 180°.
- So find the exterior angle first, then subtract.
- It turns a formula into a single division.
- It also works backwards: given the interior angle, find n.

```
regular octagon
exterior 360 ÷ 8 = 45°
interior 180 − 45 = 135°
```

Working backwards

- Given an interior angle, subtract from 180 to get the exterior.
- Then n = 360 ÷ exterior.
- If that is not a whole number, no such regular polygon exists.
- That last check is worth a mark on its own.

```
interior 150°
exterior 30°
n = 360 ÷ 30 = 12 → dodecagon
```

8.2 The parts of a circle

The three distances

- The **radius** goes from the centre to the edge.
- The **diameter** goes right across, through the centre.
- The **circumference** goes all the way round the outside.
- Almost every circle question is about one of these three.

```
centre → edge radius
edge → centre → edge diameter
all the way round circumference
```

d = 2r

- A diameter is two radii laid end to end.
- So the diameter is twice the radius.
- And the radius is half the diameter.
- This one relationship answers most of the fluency questions.

```
r = 7 cm → d = 14 cm
d = 26 cm → r = 13 cm
```

Chords and arcs

- A **chord** joins any two points on the circle.
- A diameter is the special chord that passes through the centre.
- It is also the longest chord there is.
- An **arc** is a piece of the circumference.

```
every diameter is a chord ✓
every chord is a diameter ✗
```

A circle is perfectly symmetrical

- Every line through the centre is a line of symmetry.
- There are infinitely many such lines.
- Any rotation about the centre leaves the circle unchanged.
- So its order of rotational symmetry is infinite too.

```
lines of symmetry: infinitely many
rotational order: infinite
```

Reading the question

- “How far from the centre?” — that is the radius.
- “How wide across?” — that is the diameter.
- “How far round?” — that is the circumference.
- Half the marks in this section are for reading, not calculating.

```
a wheel 70 cm across → d = 70, r = 35
a pond 15 m from the middle to the edge → r = 15, d = 30
```

Circles that touch

- When two circles touch, the distance between their centres is r + r.
- A circle fitting exactly inside a square has d equal to the square's side.
- Draw the picture and mark the radii — the answer usually appears.
- These are the questions that separate grades.

```
two circles r = 7 touching → centres 14 cm apart
circle r = 9 inside a square → side 18 cm
```

8.3 Symmetry

Line symmetry

- Fold the shape along the line.
- If the two halves match exactly, it is a line of symmetry.
- A shape can have none, one, or many.
- Imagining the fold is more reliable than judging by eye.

```
rectangle 2 lines
equilateral triangle 3 lines
parallelogram 0 lines
circle infinitely many
```

Rotational symmetry

- Turn the shape about its centre.
- Count how many times it looks the same in one full turn.
- That count is the **order**.
- The smallest turn that works is 360 ÷ order.

```
square: same at 90°, 180°, 270°, 360°
order 4, smallest turn 90°
```

Order 1 means none

- Every shape looks the same after a full 360° turn.
- So the order is never 0 — the smallest possible order is 1.
- Order 1 is how we say “no rotational symmetry”.
- This is the commonest slip in the topic.

```
scalene triangle → order 1
trapezium → order 1
✗ never write order 0
```

Regular polygons

- A regular n-gon has exactly n lines of symmetry.
- Its order of rotational symmetry is also n.
- The smallest turn is 360 ÷ n.
- One number tells you everything about its symmetry.

```
regular pentagon: 5 lines, order 5, 72°
regular octagon: 8 lines, order 8, 45°
```

The two are independent

- A shape can have rotational symmetry with no lines at all.
- The parallelogram is the standard example: order 2, no fold line.
- So always check both separately.
- Never assume one from the other.

```
parallelogram lines 0 order 2
kite lines 1 order 1
rhombus lines 2 order 2
```

Symmetry around us

- Letters, signs, tiles, flowers and wheels are full of it.
- Count the lines by looking for fold lines.
- Count the order by imagining the turn.
- Real objects are the easiest place to practise.

```
letter H: 2 lines, order 2
letter S: 0 lines, order 2
5-spoke wheel: 5 lines, order 5
```

8.4 3D shapes and nets

Faces, edges and vertices

- A **face** is a flat surface — what you would paint.
- An **edge** is a line where two faces meet — what you would trace.
- A **vertex** is a corner where edges meet.
- Count them systematically, not at random: top, bottom, sides.

```
cube: 6 faces, 12 edges, 8 vertices
tetrahedron: 4 faces, 6 edges, 4 vertices
```

Prisms

- A prism has the same cross-section all the way along.
- Two identical ends, joined by rectangles.
- One rectangle for each side of the end shape.
- So an n-gon prism has n + 2 faces.

```
triangular prism: 2 + 3 = 5 faces
hexagonal prism: 2 + 6 = 8 faces
edges: 3n vertices: 2n
```

Pyramids

- A pyramid has a base and faces rising to a single apex.
- The base can be any polygon.
- One triangular face for each side of the base.
- So an n-gon pyramid has n + 1 faces and n + 1 vertices.

```
square-based: 1 + 4 = 5 faces, 5 vertices, 8 edges
hexagonal: 1 + 6 = 7 faces, 7 vertices, 12 edges
```

Nets

- Cut a box along its edges and flatten it.
- The flat shape is a **net**.
- Every face of the solid appears exactly once.
- There is usually more than one correct net.

```
a cube has 11 different nets
each has 6 squares, joined edge to edge
```

Euler's rule

- For any solid with flat faces and no holes:
- **faces + vertices − edges = 2**.
- It works for every prism and every pyramid.
- Use it to check a count, or to find a missing one.

```
cube: 6 + 8 − 12 = 2 ✓
tetrahedron: 4 + 4 − 6 = 2 ✓
8 faces, 12 vertices → E = 18
```

Counting without a picture

- Prism on an n-gon: n + 2 faces, 3n edges, 2n vertices.
- Pyramid on an n-gon: n + 1 faces, 2n edges, n + 1 vertices.
- Derive them by thinking about the ends and the sides.
- Then check with Euler's rule before you write the answer.

```
decagonal prism: 12 faces, 30 edges, 20 vertices
check: 12 + 20 − 30 = 2 ✓
```