

Name _____

Date _____

Unit 7 · Fractions — teaching slides

30 slides, formatted for printing.

7.1 Fractions and equivalence

What the two numbers mean

- The **denominator** tells you how many equal parts the whole is cut into.
- The **numerator** tells you how many of those parts you have.
- A bigger denominator means *smaller* parts — that is the surprise.
- Which is why $1/8$ is less than $1/2$, even though $8 > 2$.

$3/4 \rightarrow 4$ equal parts, you have 3
 $1/8 \rightarrow 8$ equal parts, you have 1
 $1/8 < 1/2$ because eighths are smaller than halves

Equivalent fractions

- Multiply top **and** bottom by the same number, and the value is unchanged.
- Divide top and bottom by the same number, and again it is unchanged.
- Adding the same number to both does **not** work — it changes the value.
- This is the one rule that everything else in the unit is built on.

$1/2 = 2/4 = 3/6 = 50/100$ ✓
 $1/2 \rightarrow$ add 1 to each $\rightarrow 2/3 \times$ different value

Simplest form

- Divide both parts by their highest common factor.
- Using the HCF gets you there in one step.
- Using a smaller factor works too — you just have to go again.
- Cambridge expects the simplest form unless it says otherwise.

$18/24$ HCF is 6
 $18 \div 6 = 3$, $24 \div 6 = 4$
 $18/24 = 3/4$

Comparing fractions

- Never compare numerators while the denominators differ.
- Rewrite both over a common denominator — the LCM is tidiest.
- Now the parts are the same size, so the numerators can be compared.
- Write the conclusion in terms of the original fractions.

$\frac{2}{3}$ or $\frac{3}{5}$?
 LCM of 3 and 5 is 15
 $\frac{2}{3} = \frac{10}{15}$, $\frac{3}{5} = \frac{9}{15}$
 $10 > 9$, so $\frac{2}{3}$ is larger

Improper fractions and mixed numbers

- An **improper** fraction is at least 1: the top is at least the bottom.
- A **mixed number** pulls the whole part out in front.
- To convert: divide, and the remainder stays as the fraction.
- To go back: multiply the whole by the denominator, then add the top.

$\frac{7}{4} \rightarrow 7 \div 4 = 1 \text{ r } 3 \rightarrow 1 \frac{3}{4}$
 $2 \frac{1}{3} \rightarrow 2 \times 3 + 1 = 7 \rightarrow \frac{7}{3}$

Fractions as decimals

- A fraction is a division: $\frac{3}{4}$ means $3 \div 4$.
- Some divide exactly and some do not.
- Learn the common ones by heart — they save time all year.
- Knowing them also makes comparing much faster.

$\frac{1}{2} = 0.5$ $\frac{1}{4} = 0.25$
 $\frac{3}{4} = 0.75$ $\frac{1}{5} = 0.2$
 $\frac{1}{8} = 0.125$ $\frac{1}{10} = 0.1$
 $\frac{1}{3} = 0.333\dots$ does not stop

7.2 Adding and subtracting fractions

Why denominators must match

- Adding fractions means counting parts of the same size.
- Quarters and thirds are different sizes, so they cannot be counted together.
- Rewriting both over a common denominator makes the parts the same size.
- Then, and only then, you add the numerators.

$\frac{1}{2} + \frac{1}{3}$ parts are different sizes
 $= \frac{3}{6} + \frac{2}{6}$ now both are sixths
 $= \frac{5}{6}$

The method

- Find the LCM of the denominators.
- Rewrite each fraction with that denominator.
- Add or subtract the numerators. The denominator stays put.
- Simplify, and convert to a mixed number if that suits the question.

$$\begin{aligned} & 3/4 + 1/6 \\ & \text{LCM of 4 and 6 is 12} \\ & = 9/12 + 2/12 \\ & = 11/12 \end{aligned}$$

The mistake everyone makes once

- Adding the denominators as well as the numerators.
- $1/2 + 1/3$ is not $2/5$.
- Check by size: the answer must be bigger than $1/2$, and $2/5$ is not.
- Estimating before you calculate catches this every time.

$$\begin{aligned} & \times 1/2 + 1/3 = 2/5 \\ & \checkmark 1/2 + 1/3 = 3/6 + 2/6 = 5/6 \\ & 5/6 > 1/2 \checkmark \text{ sensible} \end{aligned}$$

Subtracting

- Exactly the same first step: a common denominator.
- Then subtract the numerators.
- If the first numerator is smaller, the answer is negative — that is allowed.
- Simplify at the end, not in the middle.

$$\begin{aligned} & 3/4 - 1/6 \\ & = 9/12 - 2/12 \\ & = 7/12 \end{aligned}$$

Mixed numbers: adding

- Route 1: add the whole parts, add the fraction parts, then combine.
- Route 2: convert both to improper fractions and add.
- Both are correct — pick one and be consistent.
- If the fraction parts make more than 1, carry into the whole number.

$$\begin{aligned} & 1 \frac{1}{2} + 2 \frac{1}{4} \\ & \text{wholes: } 1 + 2 = 3 \\ & \text{fractions: } 1/2 + 1/4 = 3/4 \\ & = 3 \frac{3}{4} \end{aligned}$$

Mixed numbers: subtracting

- Here the two routes are not equally safe.
- Subtracting the parts separately can need borrowing from the whole.
- Converting to improper fractions never does.
- Use improper fractions for subtraction and save yourself the trouble.

$$\begin{aligned} 3 \frac{1}{4} - 1 \frac{3}{4} \\ = \frac{13}{4} - \frac{7}{4} \\ = \frac{6}{4} = \frac{3}{2} = 1 \frac{1}{2} \end{aligned}$$

7.3 Multiplying fractions

The rule

- Multiply the numerators together.
- Multiply the denominators together.
- Simplify at the end.
- That is the whole method — it is easier than adding.

$$\begin{aligned} \frac{2}{3} \times \frac{3}{4} \\ \text{tops: } 2 \times 3 = 6 \\ \text{bottoms: } 3 \times 4 = 12 \\ = \frac{6}{12} = \frac{1}{2} \end{aligned}$$

“Of” means multiply

- “Half of six” is $\frac{1}{2} \times 6$.
- “Two thirds of the class” is $\frac{2}{3} \times$ the number in the class.
- So every fraction-of-a-quantity question is a multiplication.
- Read the word “of” as a $\times$ sign and the question becomes routine.

$$\begin{aligned} \frac{3}{4} \text{ of } 20 \\ = \frac{3}{4} \times 20 \\ = \frac{60}{4} = 15 \end{aligned}$$

Why the answer gets smaller

- Multiplying by a number less than 1 makes things smaller.
- Half of a half is a quarter.
- This feels wrong at first because whole-number multiplying always grows.
- Use it as a check: if your answer got bigger, look again.

$$\begin{aligned} \frac{1}{2} \times \frac{1}{2} &= \frac{1}{4} \\ \frac{1}{2} \times 8 &= 4 \\ \text{both smaller than what you started with} \end{aligned}$$

Cancel first

- Look for a factor shared by a numerator and a denominator.
- Divide both by it before you multiply.
- The arithmetic stays small and the answer arrives simplified.
- This is the same idea as simplifying, done early instead of late.

$$\begin{aligned} & 3/4 \times 8/9 \\ & 3 \text{ and } 9 \text{ share } 3; 4 \text{ and } 8 \text{ share } 4 \\ & \rightarrow 1/1 \times 2/3 = 2/3 \end{aligned}$$

Multiplying by a whole number

- A whole number is a fraction over 1: 8 is $8/1$.
- So $3/4 \times 8 = 24/4 = 6$.
- Quicker in your head: divide by the bottom, multiply by the top.
- $8 \div 4 = 2$, then $2 \times 3 = 6$.

$$\begin{aligned} & 3/4 \times 8 \\ & 8 \div 4 = 2 \\ & 2 \times 3 = 6 \end{aligned}$$

Mixed numbers

- Convert to improper fractions first.
- Then multiply exactly as usual.
- Convert back at the end if the question was in mixed numbers.
- Do not multiply whole parts and fraction parts separately — that fails.

$$\begin{aligned} & 1 \frac{1}{2} \times \frac{2}{3} \\ & = \frac{3}{2} \times \frac{2}{3} \\ & = \frac{6}{6} = 1 \end{aligned}$$

7.4 Dividing fractions

What dividing really asks

- $6 \div 2$ asks: how many 2s fit into 6?
- $6 \div 1/2$ asks: how many halves fit into 6?
- Twelve. So dividing by a half doubled the number.
- Once you read division this way, the fraction rules stop being strange.

$$\begin{aligned} & 6 \div 2 = 3 \\ & 6 \div 1/2 = 12 \\ & \text{small divisor} \rightarrow \text{big answer} \end{aligned}$$

The reciprocal

- Turn a fraction upside down and you have its reciprocal.
- A number times its reciprocal is always 1.
- A whole number n is $n/1$, so its reciprocal is $1/n$.
- This one idea turns every division into a multiplication.

$$\begin{aligned} 3/5 &\rightarrow 5/3 & 3/5 \times 5/3 &= 1 \\ 4 &\rightarrow 1/4 & 4 \times 1/4 &= 1 \\ 1/8 &\rightarrow 8 \end{aligned}$$

Keep, flip, multiply

- Keep the first fraction as it is.
- Flip the second one.
- Change $\div$ to $\times$, and multiply as usual.
- Simplify at the end.

$$\begin{aligned} 3/4 \div 1/2 \\ &= 3/4 \times 2/1 \\ &= 6/4 = 3/2 = 1 \frac{1}{2} \end{aligned}$$

Dividing by a whole number

- Dividing by 3 is multiplying by $1/3$.
- So the denominator gets multiplied and the numerator stays.
- $2/3 \div 4 = 2/12 = 1/6$.
- Sharing a fraction between people is exactly this.

$$\begin{aligned} 2/3 \div 4 \\ &= 2/3 \times 1/4 \\ &= 2/12 = 1/6 \end{aligned}$$

Why the answer grows

- Dividing by something less than 1 makes the answer bigger.
- This is the exact mirror of multiplying by a fraction.
- Use it as a check on every answer you write.
- If you divided by a half and the answer shrank, you flipped the wrong one.

$$\begin{aligned} 8 \div 1/4 &= 32 \text{ bigger } \checkmark \\ 8 \times 1/4 &= 2 \text{ smaller } \checkmark \end{aligned}$$

Mixed numbers

- Convert to improper fractions first — always.
- Then keep, flip, multiply.
- Convert back if the question used mixed numbers.
- Never try to divide the whole parts and fraction parts separately.

$$\begin{aligned} 2 \frac{1}{4} \div 3 \\ = \frac{9}{4} \div 3 \\ = \frac{9}{4} \times \frac{1}{3} = \frac{9}{12} = \frac{3}{4} \end{aligned}$$

7.5 Making fraction calculations easier

Look before you calculate

- Every calculation can be done more than one way.
- The answer is the same; the effort is not.
- Spending five seconds choosing the order saves a minute of arithmetic.
- It also cuts the number of places where a slip can happen.

$$\begin{aligned} \frac{3}{4} \text{ of } 48 \\ 48 \times 3 = 144, \text{ then } \div 4 = 36 \\ 48 \div 4 = 12, \text{ then } \times 3 = 36 \leftarrow \text{kinder} \end{aligned}$$

Cancel before multiplying

- Find a factor shared by any numerator and any denominator.
- Divide both by it before multiplying.
- The answer comes out already in its simplest form.
- This is simplifying done early rather than late.

$$\begin{aligned} \frac{4}{9} \times \frac{3}{8} \\ 4 \text{ and } 8 \text{ share } 4; 3 \text{ and } 9 \text{ share } 3 \\ = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6} \end{aligned}$$

Divide first, then multiply

- For “a fraction of a quantity”, divide by the denominator first.
- Then multiply by the numerator.
- The numbers stay small enough to do in your head.
- It works because multiplication and division may be done in any order.

$$\begin{aligned} \frac{5}{6} \text{ of } 72 \\ 72 \div 6 = 12 \\ 12 \times 5 = 60 \end{aligned}$$

Reorder an addition

- Addition can be done in any order.
- Look for a pair that makes a whole number.
- Add those first, then bring in the rest.
- Careful: subtraction and division cannot be reordered.

$$\begin{aligned}1/4 + 1/2 + 3/4 \\ 1/4 + 3/4 &= 1 \\ 1 + 1/2 &= 1 \frac{1}{2}\end{aligned}$$

Spot the reciprocals

- If one fraction is another turned upside down, the product is 1.
- No multiplying, no simplifying, no working.
- The same pattern makes a division collapse to 1.
- Train your eye for it — it appears in exams on purpose.

$$\begin{aligned}5/8 \times 8/5 &= 1 \\ 2/7 \times 7/2 &= 1 \\ 3/4 \div 3/4 &= 1\end{aligned}$$

Estimate as a safety net

- Before calculating, decide roughly what the answer should be.
- Is it bigger or smaller than what you started with?
- Is it near a half, near a whole?
- An answer that fails the estimate is wrong, whatever the working says.

$$\begin{aligned}3/4 \text{ of } 48: & \text{ about three quarters of } 50 \rightarrow \text{ near } 36 \checkmark \\ 6 \div 1/2: & \text{ dividing by a small piece } \rightarrow \text{ much more than } 6\end{aligned}$$