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Name _____ Date _____

Unit 2 · Expressions, formulae and equations — teaching slides

36 slides, formatted for printing.

2.1 Constructing expressions

Letters stand for numbers

- When you do not know a number, use a letter for it.
- The letter is called the **variable** or the **unknown**.
- A bag holds an unknown number of sweets — call it n.

```
n sweets in the bag
Take 3 out → n − 3 sweets left
Put 2 in → n + 2 sweets
```

Expression or equation?

- An **expression** contains numbers and letters but **no** equals sign.
- An **equation** contains an equals sign.
- This distinction matters — you simplify expressions, you solve equations.

```
5n + 4 is an expression
5n + 4 = 19 is an equation
```

The words of algebra

- A **term** is one part of an expression.
- The **coefficient** is the number multiplying the letter.
- A **constant** is a number on its own.

```
In 5n + 4 there are two terms: 5n and 4
In 5n the coefficient of n is 5
In 5n + 4 = 19 the constants are 4 and 19
```

How to write algebra correctly

- Number first, then the letter.
- Never write the 1.
- Division becomes a fraction.
- Repeated multiplication becomes an index.

```
x × 3 → 3x (not x3)
1 × a → a (not 1a)
y ÷ 5 → y/5
a × a → a² a × a × a → a³
```

Turning words into expressions

- “more than” → add. “less than” → subtract.
- Watch the order: **x less than y** is y − x.
- “doubled” → × 2, “tripled” → × 3, “half of” → ÷ 2.

```
x more than y → y + x
x less than y → y − x
m more than two times n → 2n + m
half of x → x/2
```

Order of operations still applies

- Brackets change the meaning, so put them in when the words need them.
- “4 more than double n” is not the same as “double the number 4 more than n”.

```
4 more than double n → 2n + 4
double (4 more than n) → 2(n + 4) = 2n + 8
Check with n = 3: 10 versus 14 — different!
```

2.2 Using expressions and formulae

What is a formula?

- A formula is a rule linking quantities, written with an equals sign.
- You can write it in words or in letters.
- The letters are called **variables** because their values change.

```
Area of rectangle = length × width
A = l × w (or just A = lw)
```

Substituting numbers

- Replace each letter with its value.
- Then work out the answer using the normal order of operations.
- Write the substitution down before you calculate — it prevents slips.

```
A = lw with l = 5, w = 4 → A = 5 × 4 = 20 cm²
3x + 2 with x = 4 → 3 × 4 + 2 = 14
```

Order of operations matters

- Do the multiplying before the adding.
- Do what is inside brackets first.
- Indices come before multiplication.

```
3x + 2 with x = 4 → 12 + 2 = 14 (not 3 × 6 = 18)
4(y + 1) with y = 6 → 4 × 7 = 28
x² + 1 with x = 5 → 25 + 1 = 26
```

Substituting negative numbers

- Put brackets round a negative value — it stops sign mistakes.
- Squaring a negative gives a positive.

```
5x − 2 with x = −3 → 5(−3) − 2 = −15 − 2 = −17
n² with n = −4 → (−4)² = 16
3d + 8 with d = −5 → −15 + 8 = −7
```

Writing your own formula

- Decide what changes — those become your letters.
- Choose letters that remind you of the quantity.
- Always say what each letter stands for.

```
n books at $8 each → C = 8n
$40 call-out plus $25 an hour → T = 25h + 40
Say: T is the total charge in dollars, h is the number of hours
```

Working backwards

- If you know the answer, you can find the missing value.
- Substitute what you know, then solve the equation that is left.

```
T = 25h + 40 and T = 190
25h + 40 = 190 → 25h = 150 → h = 6 hours
```

2.3 Collecting like terms

Bricks and like terms

- Three red bricks of length x cm have total length 3x cm.
- Two blue bricks of length y cm have total length 2y cm.
- Together they are 3x + 2y — and that is as far as you can go.
- You cannot combine terms with different letters.

```
x + x + x = 3x
y + y = 2y
3x + 2y stays as 3x + 2y
```

What makes terms 'like'?

- Like terms have **exactly** the same letters to the same powers.
- 3x and 5x are like terms.
- 3x and 3y are not — different letters.
- 3x and $3x^2$ are not — different powers.

```
5a and 2a ✓ like
4p and 4 ✗ not like
2m and m² ✗ not like
2ab and 5ab ✓ like
```

Collecting like terms

- Go through the expression one letter at a time.
- Add or subtract the coefficients; the letter stays the same.
- Do the plain numbers last.
- Keep the sign that is in front of each term.

```
4x + 3y + 2x − y
x terms: 4x + 2x = 6x
y terms: 3y − y = 2y
Answer: 6x + 2y
```

Writing algebra properly

- These conventions are marked in Cambridge assessments.
- Left column is right, right column is wrong.

```
a ✓ 1a, 1 × a, a¹ ✗
2a ✓ a + a, a², a2 ✗
ab ✓ a × b, ba ✗
a² ✓ a × a ✗
a³ ✓ a × a × a ✗
```

Negative coefficients

- Subtracting more than you have is fine in algebra.
- Keep the minus sign with the term that follows it.

```
4a − 7a = −3a
6y − y − 2y = 3y
5x + 7 − 2x − 3 = 3x + 4
```

Fraction terms

- Treat them exactly like ordinary fractions.
- Find a common denominator, then add or subtract the numerators.
- Simplify the fraction at the end.

```
3a/4 + 5a/12
= 9a/12 + 5a/12
= 14a/12 = 7a/6
```

2.4 Expanding brackets

What expanding means

- 4(n + 3) is short for 4 × (n + 3).
- The 4 multiplies **everything** inside the brackets.
- Expanding is sometimes called “multiplying out”.

```
4(n + 3) = 4 × n + 4 × 3
= 4n + 12
```

The commonest mistake

- Learners multiply the first term and forget the second.
- Draw arrows from the outside term to each inside term.
- Count the terms inside — that is how many multiplications you do.

```
4(n + 3) = 4n + 3 ✗ wrong
4(n + 3) = 4n + 12 ✓ right
```

Watch the signs

- The sign belongs to the term that follows it.
- A minus inside stays a minus after multiplying by a positive.

```
2(x − 5) = 2 × x − 2 × 5 = 2x − 10
7(2 − d) = 14 − 7d
```

Three terms inside

- The method does not change — just do three multiplications.
- Work along the bracket in order and keep each sign.

```
3(2g + h − 7)
= 3 × 2g + 3 × h − 3 × 7
= 6g + 3h − 21
```

A letter outside the brackets

- The rule is identical when the multiplier is a letter.
- Remember $x \times x = x^2$.
- This is how you find the area of a rectangle with algebraic sides.

```
x(x + 3) = x² + 3x
2x(x + 4) = 2x² + 8x
```

Expand, then collect

- With two brackets, expand both first.
- Then collect the like terms.
- A minus in front of the second bracket changes both of its signs.

```
2(x + 3) + 3(x + 1) = 2x + 6 + 3x + 3 = 5x + 9
5(y + 3) − 2(y + 1) = 5y + 15 − 2y − 2 = 3y + 13
```

2.5 Constructing and solving equations

An equation is a balance

- The equals sign says the two sides have the same value.
- Whatever you do to one side, do to the other, and it stays balanced.
- Solving means getting the letter alone on one side.

```
x + 5 = 12
x + 5 − 5 = 12 − 5
x = 7
```

Inverse operations

- Addition and subtraction undo each other.
- Multiplication and division undo each other.
- Look at what has been done to the letter, then undo it.

```
x − 3 = 12 → add 3 → x = 15
3n = 18 → divide by 3 → n = 6
m/4 = 5 → multiply by 4 → m = 20
```

Two-step equations

- Undo in the reverse order to the way the expression was built.
- Deal with the + or − first, then the × or ÷.

```
2y + 4 = 16
2y = 16 − 4 = 12 (subtract 4)
y = 12 ÷ 2 = 6 (divide by 2)
```

Equations with brackets

- Either divide by the number outside first,
- or expand the brackets and then solve.
- Both routes give the same answer — choose the tidier one.

```
4(x − 2) = 20
x − 2 = 5 (divide by 4)
x = 7
Or: 4x − 8 = 20 → 4x = 28 → x = 7
```

Always check

- Put your answer back into the **original** equation.
- If both sides are equal, your solution is right.
- This catches almost every arithmetic slip.

```
Solve 2y + 4 = 16 → y = 6
Check: 2 × 6 + 4 = 12 + 4 = 16 ✓
```

Building an equation from words

- Choose a letter for the unknown and say what it stands for.
- Translate the sentence one phrase at a time.
- Solve, then answer the question that was actually asked.

```
“I think of a number, double it and add 3; the answer is 17.”
2n + 3 = 17
2n = 14, so n = 7
```

2.6 Inequalities

The two symbols

- **<** means “is less than”.
- **>** means “is greater than”.
- The wide end of the symbol faces the **larger** quantity.
- Read the statement from left to right.

```
x < 10 reads “x is less than 10”
x > 10 reads “x is greater than 10”
x < −4 reads “x is less than −4”
```

Showing an inequality on a number line

- Draw an **open circle** at the boundary value.
- The circle is open because that value is not included.
- Draw an arrow: right for >, left for <.

```
x > 5: open circle at 5, arrow pointing right
y < 2: open circle at 2, arrow pointing left
```

The smallest or largest integer

- Remember an integer is a whole number.
- For $x > 5$ the boundary 5 is excluded, so the smallest integer is 6.
- For $y < 2$ the boundary 2 is excluded, so the largest integer is 1.
- With negatives, count carefully along the number line.

```
x > 5 → smallest integer 6
y < 2 → largest integer 1
x > −3 → smallest integer −2
y < −4 → largest integer −5
```

Listing the values

- The list of solutions never ends.
- Write the first few, then three dots to show it continues.
- Start from the value closest to the boundary.

```
x > 5: 6, 7, 8, 9, 10, …
y < 2: 1, 0, −1, −2, −3, …
```

Two inequalities together

- Two inequalities can trap a value between two numbers.
- Now the list is finite, so you can write all of it.
- Check both ends carefully — neither boundary is included.

```
x > 2 and x < 7
Integers: 3, 4, 5, 6 (four values)
x > −2 and x < 4 → −1, 0, 1, 2, 3
```

Solving an inequality

- Use exactly the same steps as for an equation.
- Do the same thing to both sides.
- The answer is a range of values, not a single number.

```
3x + 1 > 16
3x > 15 (subtract 1)
x > 5 (divide by 3)
Smallest integer: 6
```