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Unit 15 · Shapes, area and volume — teaching slides

36 slides, formatted for printing.

15.1 Perimeter and area of rectangles

Perimeter and area are different things

- Perimeter is the distance all the way round the edge.
- Area is the amount of surface inside.
- Perimeter is measured in cm; area in cm².
- The unit tells you which one you have found.

```
rectangle 8 cm by 5 cm
perimeter 8 + 5 + 8 + 5 = 26 cm
area 8 × 5 = 40 cm²
```

Why area is measured in squares

- A square centimetre is a square 1 cm by 1 cm.
- The area is how many of them fit inside.
- In an 8 by 5 rectangle there are 5 rows of 8.
- So the formula is a multiplication, not an addition.

```
5 rows of 8 squares
5 × 8 = 40
area 40 cm²
```

The two formulas

- Perimeter of a rectangle = 2(l + w).
- Area of a rectangle = l × w.
- For a square, both sides are the same.
- Perimeter = 4s and area = s².

```
l = 12, w = 7
P = 2 × (12 + 7) = 38 cm
A = 12 × 7 = 84 cm²
```

Working backwards

- Given the area and one side, divide.
- Given the perimeter and one side, subtract then halve.
- Write the formula down first, then rearrange it.
- Check the answer by putting it back in.

```
area 48, length 8
48 ÷ 8 = 6 cm wide
perimeter 30, length 9
30 − 18 = 12, then 12 ÷ 2 = 6 cm
```

Congruent shapes

- Congruent means same shape and same size.
- Every corresponding side is equal.
- Every corresponding angle is equal.
- A translation, reflection or rotation gives a congruent shape.

```
two 6 cm by 4 cm rectangles: congruent
a 6 by 4 and a 12 by 2: equal area, not congruent
```

Equal area does not mean equal shape

- Many different rectangles share an area.
- 24 cm² can be 1 × 24, 2 × 12, 3 × 8 or 4 × 6.
- Their perimeters are all different.
- So area alone never identifies a shape.

```
1 × 24 perimeter 50 cm
2 × 12 perimeter 28 cm
3 × 8 perimeter 22 cm
4 × 6 perimeter 20 cm
```

15.2 Compound shapes

What a compound shape is

- It is made from rectangles joined together.
- So nothing new is needed — just the rectangle formula.
- The skill is choosing how to break it up.
- And keeping track of the side lengths.

```
an L-shape
= two rectangles
or one rectangle with a bite out
```

Method 1: split it up

- Draw one straight line to make two rectangles.
- Work out the missing side lengths by subtracting.
- Find each area.
- Add them.

```
lower 10 × 4 = 40
upper 6 × 3 = 18
total 40 + 18 = 58 cm²
```

Method 2: subtract

- Draw the smallest rectangle the shape fits inside.
- Find its area.
- Find the area of the missing piece.
- Subtract.

```
whole 10 × 7 = 70
bite 4 × 3 = 12
area 70 − 12 = 58 cm²
```

Both methods, one answer

- The area of a shape does not depend on how you cut it.
- So the two methods must agree.
- If they do not, one of them has an arithmetic slip.
- This is the cheapest check in the whole unit.

```
split: 40 + 18 = 58
subtract: 70 − 12 = 58
agreed
```

Perimeter is a walk round the outside

- Start at one corner and go all the way round.
- Add every edge you walk along.
- Do not include the line you drew to split the shape.
- It is inside, not on the edge.

```
10 + 4 + 4 + 3 + 6 + 7
= 34 cm
```

A surprise about corners

- Cut a rectangular corner out of a rectangle.
- The area goes down.
- But the perimeter stays exactly the same.
- The two new edges are as long as the two they replaced.

```
10 by 7 rectangle: perimeter 34 cm
with a 4 by 3 corner gone: still 34 cm
area 70 cm² → 58 cm²
```

15.3 Units of area

The factor gets squared

- 1 cm = 10 mm. That is a length conversion.
- An area is a length times a length.
- So the factor is used twice.
- 1 cm² = 10 × 10 = 100 mm².

```
length: × 10
area: × 10 × 10 = × 100
1 cm² = 100 mm²
```

Picture it

- Draw a square 1 cm by 1 cm.
- Rule it into millimetre squares.
- There are 10 rows of 10.
- So 100 of them — nothing to memorise.

```
10 rows × 10 squares
= 100 mm² in 1 cm²
```

The metre and the kilometre

- 1 m = 100 cm, so 1 m² = 100 × 100 = 10 000 cm².
- 1 km = 1000 m, so 1 km² = 1000 × 1000 = 1 000 000 m².
- The zeros double every time.
- Anyone who answers 1000 m² for 1 km² has forgotten to square.

```
1 m² = 10 000 cm²
1 km² = 1 000 000 m²
```

The hectare

- Land is measured in hectares.
- A hectare is a square 100 m by 100 m.
- So 1 ha = 10 000 m².
- And 1 km² = 100 ha.

```
1 ha = 10 000 m²
roughly two football pitches
1 km² = 100 ha
```

Which way round?

- Big unit to small unit: multiply.
- Small unit to big unit: divide.
- Many small squares fill one big square.
- If the answer looks the wrong size, you divided instead of multiplied.

```
3 m² → cm²: 3 × 10 000 = 30 000
50 000 m² → ha: 50 000 ÷ 10 000 = 5
```

Convert before you multiply

- A shape may be given in mixed units.
- Put both sides into the same unit first.
- Then use the area formula once.
- Converting afterwards is where mistakes happen.

```
2 m by 50 cm
= 2 m by 0.5 m
area 1 m²
```

15.4 Area of a triangle

Deriving the formula

- Take any triangle.
- Make a second copy and turn it upside down.
- The two fit together into a parallelogram.
- That parallelogram is base × height, so the triangle is half.

```
parallelogram = b × h
triangle = ½ × b × h
b = 8, h = 5
8 × 5 = 40, then 40 ÷ 2 = 20 cm²
```

Which height?

- The height is measured at right angles to the base.
- Not along a sloping side — that is longer.
- The diagram marks it with a dashed line and a right angle.
- Using a slanted side is the standard error here.

```
base 12 cm
sloping side 13 cm ✗
perpendicular height 5 cm ✓
```

When the height falls outside

- An obtuse triangle leans past its base.
- The perpendicular height then lands outside the shape.
- Extend the base with a dashed line and measure to that.
- The formula does not change at all.

```
base 12 cm, height 7 cm
12 × 7 = 84
84 ÷ 2 = 42 cm²
```

Same base, same height, same area

- Slide the apex sideways, keeping its height.
- The triangle looks completely different.
- But the base and the height are unchanged.
- So the area is unchanged too.

```
right, acute or obtuse
base 10, height 6
area 30 cm² in every case
```

Working backwards

- Given the area and the base, find the height.
- Undo the halving first: double the area.
- Then divide by the base.
- Reverse order, exactly as with any formula.

```
area 24, base 8
24 × 2 = 48
48 ÷ 8 = 6 cm
```

Parallelograms and compound shapes

- A parallelogram is two of those triangles.
- So its area is base × height, with no halving.
- A house shape is a rectangle plus a triangle.
- Find each area and add — the method from 15.2 again.

```
rectangle 10 × 6 = 60
triangle 10 × 4 ÷ 2 = 20
total 80 cm²
```

15.5 Volume of cubes and cuboids

What volume is

- Volume is the space a solid takes up.
- It is counted in cubes 1 cm on each side.
- A 5 by 3 base holds 15 cubes in one layer.
- Four layers make 60 cubes altogether.

```
5 × 3 = 15 in a layer
4 layers
15 × 4 = 60 cm³
```

The formula

- Volume of a cuboid = length × width × height.
- Three dimensions, three factors.
- For a cube all three are equal, so it is s^3.
- The units are cm^3 because three lengths were multiplied.

```
5 × 3 × 4 = 60 cm³
cube of edge 4: 4 × 4 × 4 = 64 cm³
```

Layers, not magic

- The base area tells you how many cubes are in one layer.
- The height tells you how many layers there are.
- So volume = base area × height.
- This way of seeing it works for every prism, not just cuboids.

```
base 8 × 5 = 40 cm²
height 3 cm
volume 40 × 3 = 120 cm³
```

Cubic units and litres

- 1 litre is 1000 cm^3.
- That is a cube 10 cm by 10 cm by 10 cm.
- 1 m^3 is 100 × 100 × 100 = 1 000 000 cm^3.
- The factor is cubed — not squared as it was for area.

```
2000 cm³ = 2 litres
1 m³ = 1 000 000 cm³
1 m³ = 1000 litres
```

Compound solids

- Split the solid into cuboids.
- Find each volume.
- Add them.
- If a piece has been drilled out, subtract instead.

```
6 × 4 × 3 = 72
2 × 2 × 2 = 8
total 72 + 8 = 80 cm³
```

Working backwards

- Given the volume and two dimensions, divide.
- Multiply the two you know first.
- Then divide the volume by that.
- Check by multiplying all three back together.

```
volume 240, base 4 by 4
4 × 4 = 16
240 ÷ 16 = 15 cm tall
```

15.6 Surface area, nets and views

Surface area is an area

- It is the total area of all the faces.
- Think of the paper needed to wrap the box exactly.
- So the units are cm^2, not cm^3.
- Volume fills; surface area covers.

```
volume 5 × 3 × 4 = 60 cm³
surface 2(15 + 20 + 12) = 94 cm²
different questions, different units
```

Six faces in three pairs

- Front and back are the same: l × h.
- Left and right are the same: w × h.
- Top and bottom are the same: l × w.
- Add one of each, then double.

```
l = 5, w = 3, h = 4
15 + 20 + 12 = 47
47 × 2 = 94 cm²
```

The net proves it

- Open the box out flat.
- You can see all six rectangles at once.
- Nothing is hidden round the back.
- Forgetting a face is the usual mistake, and the net prevents it.

```
top and bottom
front and back
left and right
six rectangles, three sizes
```

Cubes

- All six faces are identical squares.
- So the surface area is $6s^2$.
- And the volume is s^3.
- Keep the two apart: one is squared, one is cubed.

```
s = 4
surface 6 × 16 = 96 cm²
volume 4 × 4 × 4 = 64 cm³
```

Open boxes

- A tray or open tank has only five faces.
- Work out the closed solid first.
- Then subtract the face that is missing.
- Say in your working which face you took off.

```
12 by 8 by 5, open top
closed 392 cm²
top 12 × 8 = 96
392 − 96 = 296 cm²
```

Front, side and top views

- Look straight at the solid from each direction.
- Draw what you see, flat, with no perspective.
- For a cuboid each view is a rectangle.
- Front l × h, side w × h, top l × w.

```
cuboid 5 by 3 by 4
front 5 × 4 = 20 cm²
side 3 × 4 = 12 cm²
top 5 × 3 = 15 cm²
```