nexamind
EDUCATION

Name Date

Unit 14 · Position and transformation — teaching slides

36 slides, formatted for printing.

14.1 Coordinates and distance

Coordinates in four quadrants

- A point is fixed by two numbers, written (x, y).
- Across first, then up — always in that order.
- Either number may be negative.
- The axes divide the plane into four quadrants.

```
(3, 2) right 3, up 2
(−4, 1) left 4, up 1
(−2, −3) left 2, down 3
(5, −1) right 5, down 1
```

Numbering the quadrants

- Quadrant 1 is the top right, where both are positive.
- Then count **anticlockwise**: 2, 3, 4.
- The signs tell you the quadrant without any drawing.
- A point on an axis is in no quadrant at all.

```
quadrant 1: (+, +)
quadrant 2: (−, +)
quadrant 3: (−, −)
quadrant 4: (+, −)
```

Distance along a horizontal line

- If two points share a y-coordinate, the line is horizontal.
- Only the x has changed, so subtract the x-coordinates.
- Take the size of the answer — a distance is never negative.
- No grid, no counting squares: just arithmetic.

```
(2, 5) to (9, 5)
9 − 2 = 7
distance 7
```

Distance along a vertical line

- If two points share an x-coordinate, the line is vertical.
- Subtract the y-coordinates this time.
- Watch the negatives: 6 − (−3) = 9, not 3.
- This is Unit 1 arithmetic doing geometry work.

```
(4, −3) to (4, 6)
6 − (−3) = 9
distance 9
```

Midpoints

- The midpoint is exactly halfway along.
- Average the x-coordinates, then average the y-coordinates.
- For a horizontal or vertical line only one of them changes.
- Averaging is adding then halving — nothing new.

```
(−4, 5) and (8, 5)
(−4 + 8) ÷ 2 = 2
midpoint (2, 5)
```

Rectangles from coordinates

- Three corners of a rectangle fix the fourth.
- Opposite sides are equal and parallel.
- Once you have all four, the side lengths give perimeter and area.
- Check your fourth corner shares a coordinate with two others.

```
(1, 1), (6, 1), (6, 4)
fourth corner (1, 4)
sides 5 and 3, area 15
```

14.2 Scale drawings, maps and plans

What a scale means

- A scale drawing keeps the shape and changes only the size.
- The scale tells you the link between the two.
- “1 cm represents 5 m” is one way to write it.
- Every length on the drawing obeys the same rule.

```
1 cm represents 5 m
3 cm on the plan
3 × 5 = 15 m in real life
```

Which way round?

- Real life is bigger than the drawing.
- Drawing → real life: **multiply**.
- Real life → drawing: **divide**.
- If your answer is the wrong size, you went the wrong way.

```
1 cm to 4 m
7 cm → 7 × 4 = 28 m
36 m → 36 ÷ 4 = 9 cm
```

Ratio scales

- A scale can be written 1 : n, with no units at all.
- That works because both sides are in the same units.
- 1 : 500 means one centimetre stands for 500 centimetres.
- It equally means one metre stands for 500 metres.

```
1 : 500
1 cm → 500 cm = 5 m
so “1 cm to 5 m” and 1 : 500 are the same scale
```

Changing the units

- Most mistakes here are unit mistakes, not scale mistakes.
- 100 cm = 1 m, and 1000 m = 1 km.
- So 100 000 cm = 1 km.
- Convert last, after the scaling, and it stays simple.

```
1 : 25 000, map length 8 cm
8 × 25 000 = 200 000 cm
200 000 ÷ 100 000 = 2 km
```

Bigger number, smaller picture

- 1 : 100 shrinks real life by a factor of 100.
- 1 : 1000 shrinks it by a factor of 1000.
- So 1 : 1000 gives the smaller drawing.
- A street plan uses a small number; a country map a large one.

```
1 : 50 a room plan
1 : 25 000 a walking map
1 : 1 000 000 a country map
```

A warning about area

- Every length is multiplied by the scale factor.
- But an area is a length times a length.
- So the area is multiplied by the scale factor **squared**.
- Double the lengths and the area becomes four times as big.

```
plan 6 cm × 4 cm, 1 cm to 2 m
real 12 m × 8 m
area 24 cm² on the plan → 96 m² real (× 4, not × 2)
```

14.3 Translation

What a translation is

- A translation slides the whole shape.
- Every point moves the same distance in the same direction.
- Nothing turns, nothing flips, nothing changes size.
- The image is congruent to the object.

```
object (1, 1) (4, 1) (4, 3)
vector (−4, 1)
image (−3, 2) (0, 2) (0, 4)
```

Reading the vector

- The first number is the movement across.
- The second number is the movement up.
- Negative first number means left.
- Negative second number means down.

```
(3, 2) right 3, up 2
(−5, 1) left 5, up 1
(4, −6) right 4, down 6
```

Translating a point

- Add the vector to the coordinates.
- One addition across, one addition up.
- That is all there is to it.
- No grid, no counting squares.

```
point (−1, 5), vector (3, −2)
−1 + 3 = 2
5 + (−2) = 3
image (2, 3)
```

Finding the vector

- Given an object and its image, subtract.
- Image minus object, in each coordinate.
- Getting this the wrong way round reverses the arrows.
- Check it by applying the vector to a second vertex.

```
(2, 3) → (6, 2)
6 − 2 = 4
2 − 3 = −1
vector (4, −1)
```

One after the other

- Two translations can be replaced by one.
- Add the two vectors.
- The order does not matter — addition is commutative.
- So a chain of slides is still just a slide.

```
(4, −1) then (3, 5)
4 + 3 = 7, −1 + 5 = 4
single vector (7, 4)
```

Undoing a translation

- Reverse both parts of the vector.
- (5, 3) is undone by (−5, −3).
- The two add to (0, 0), the translation that does nothing.
- Every transformation in this unit can be undone.

```
there (5, 3)
back (−5, −3)
total (0, 0)
```

14.4 Reflection

What a reflection is

- A reflection flips the shape in a mirror line.
- Every point moves straight across the mirror.
- It ends up the same distance on the other side.
- The image is congruent to the object.

```
object at (3, 2)
mirror: the x-axis
image at (3, −2)
both 2 away from the line
```

Reflecting in the x-axis

- The x-axis is the horizontal line y = 0.
- Across is unchanged, so x stays the same.
- Up becomes down, so y changes sign.
- (x, y) → (x, −y).

```
(5, 3) → (5, −3)
(−1, 4) → (−1, −4)
(2, −6) → (2, 6)
```

Reflecting in the y-axis

- The y-axis is the vertical line x = 0.
- Up is unchanged, so y stays the same.
- Left becomes right, so x changes sign.
- (x, y) → (−x, y).

```
(5, 3) → (−5, 3)
(−1, 4) → (1, 4)
(2, −6) → (−2, −6)
```

Points on the mirror

- A point on the mirror line is 0 away from it.
- So it stays exactly where it is.
- Such points are called invariant.
- That is a useful check on a drawing.

```
(0, 4) reflected in the y-axis
it is already on the line
image (0, 4) — unmoved
```

Other mirror lines

- The mirror need not be an axis.
- x = 2 is a vertical line through 2 on the x-axis.
- y = −1 is a horizontal line through −1 on the y-axis.
- y = x is the diagonal, and reflecting in it swaps the coordinates.

```
(3, 2) in x = 1 → (−1, 2)
(5, 1) in y = 3 → (5, 5)
(3, 1) in y = x → (1, 3)
```

Finding the mirror line

- Given a shape and its image, join two matching points.
- The mirror cuts that join exactly in half.
- And it does so at right angles.
- Check with a second pair of matching points.

```
(6, 2) → (6, −2)
x unchanged, y changed sign
mirror: the x-axis
```

14.5 Rotation

What a rotation is

- A rotation turns the whole shape about a fixed point.
- That point is the centre of rotation.
- The centre is the only point that does not move.
- The image is congruent to the object.

```
centre (0, 0)
angle 90°
direction anticlockwise
all three are needed
```

A quarter turn about the origin

- Anticlockwise is the positive direction in mathematics.
- 90° anticlockwise sends (x, y) to $(-y, x)$.
- Check it on a point you can picture.
- $(3, 0)$ is on the x-axis and lands at $(0, 3)$ on the y-axis.

```
(3, 1) → (−1, 3)
(2, 4) → (−4, 2)
(−1, 3) → (−3, −1)
```

A half turn about the origin

- 180° sends (x, y) to $(-x, -y)$.
- Both signs change.
- Direction does not matter for a half turn.
- It is the same as reflecting in both axes in turn.

```
(3, 1) → (−3, −1)
(−4, 2) → (4, −2)
(5, −2) → (−5, 2)
```

Turning the other way

- 90° clockwise is the same as 270° anticlockwise.
- It sends (x, y) to (y, −x).
- Three quarter turns one way equal one the other way.
- Four quarter turns bring the shape back to the start.

```
(3, 1) → (1, −3)
(0, 5) → (5, 0)
4 × 90 = 360, a full turn
```

A centre that is not the origin

- Find the offset: how far across and up from the centre.
- Turn the offset, using the same rule as before.
- Add the centre back on.
- Three short steps, no picture needed.

```
(4, 2) about (1, 1), 90° anticlockwise
offset 3 across, 1 up
turned: 1 left, 3 up
image (0, 4)
```

Describing a rotation

- An examiner wants the angle, the direction and the centre.
- Leaving out the centre loses the mark.
- For 180° the direction may be omitted — it makes no difference.
- Check your answer by turning one vertex back again.

```
“a rotation of 90° anticlockwise about (1, 1)”
“a rotation of 180° about the origin”
```

14.6 Enlargement

What an enlargement is

- An enlargement changes the size of a shape.
- Every length is multiplied by the same scale factor.
- Every angle stays exactly as it was.
- So the shape is unchanged — only the size is different.

```
scale factor 2
3 cm → 6 cm
5 cm → 10 cm
angles unchanged
```

Enlarging from the origin

- When the centre is the origin, the rule is easy.
- Multiply both coordinates by the scale factor.
- (x, y) → (kx, ky).
- The origin itself does not move.

```
k = 2
(2, 3) → (4, 6)
(−1, 4) → (−2, 8)
```

A centre somewhere else

- Find the offset from the centre, across and up.
- Multiply that offset by the scale factor.
- Add the centre back on.
- The same three steps as a rotation about a general centre.

```
(3, 2), centre (1, 1), k = 2
offset 2 across, 1 up
doubled: 4 across, 2 up
image (5, 3)
```

Similar, not congruent

- Congruent means same shape **and** same size.
- Similar means same shape, size may differ.
- Translation, reflection and rotation give congruent images.
- Enlargement gives a similar one.

```
translate → congruent
reflect → congruent
rotate → congruent
enlarge → similar
```

What happens to area

- Every length is multiplied by k.
- An area is a length times a length.
- So an area is multiplied by k × k.
- Scale factor 3 makes a shape 9 times bigger in area.

```
4 cm × 6 cm, k = 3
12 cm × 18 cm
24 cm² → 216 cm²
24 × 9 = 216
```

Enlargement is everywhere

- A scale drawing is an enlargement.
- So is a photographic enlargement, and a model kit.
- So is a map, with a scale factor smaller than 1.
- Stage 7 uses positive whole-number scale factors only.

```
photo 5 cm wide → 20 cm wide
scale factor 20 ÷ 5 = 4
```