nexamind EDUCATION

Name Date

Unit 14 · Position and transformation — revision sheet

Everything you need from every section. Read it before the end-of-unit test.

14.1 Coordinates and distance

- Coordinates are written **(x, y)** — across first, then up. (3, 1) and (1, 3) are different points.
- The **origin** is (0, 0), where the axes cross.
- The four **quadrants** are numbered **anticlockwise from the top right**: 1 (+, +), 2 (−, +), 3 (−, −), 4 (+, −).
- A point on an axis is in **no quadrant**.
- If two points share a **y**-coordinate the line is **horizontal**: subtract the x-coordinates.
- If two points share an **x**-coordinate the line is **vertical**: subtract the y-coordinates.
- A distance is never negative — take the size of the subtraction.
- The **midpoint** of such a line is the average of the two coordinates.
- None of this needs a grid; it is arithmetic on the coordinates.

Key words: *coordinates · origin · quadrant · x-coordinate · y-coordinate · midpoint · axis*

```
(3, 2) right 3, up 2
(−4, 1) left 4, up 1
(−2, −3) left 2, down 3
(5, −1) right 5, down 1
quadrant 1: (+, +)
quadrant 2: (−, +)
quadrant 3: (−, −)
```

14.2 Scale drawings, maps and plans

• A **scale** tells you what one length on a drawing stands for in real life.
• **Drawing → real: multiply. Real → drawing: divide.**
• A scale can be written as “1 cm represents 5 m” or as the ratio **1 : 500**.
• For a ratio scale, **both sides must be in the same units**: 5 m = 500 cm, so 1 cm to 5 m is 1 : 500.
• The bigger the second number, the **smaller** the drawing.
• Useful conversions: 100 cm = 1 m, 100 000 cm = 1 km.
• Every angle is unchanged, and every length is multiplied by the same number — the shape does not distort.
• Areas do **not** scale by the same number: doubling every length multiplies the area by 4.

Key words: *scale · scale drawing · ratio scale · plan · map · represents*

```
1 cm represents 5 m
3 cm on the plan
3 × 5 = 15 m in real life
1 cm to 4 m
7 cm → 7 × 4 = 28 m
36 m → 36 ÷ 4 = 9 cm
1 : 500
```

14.3 Translation

• A **translation** slides a shape. Nothing turns and nothing flips.
• It is described by a **vector (across, up)**.
• To translate a point, **add the vector to its coordinates**.
• To find the vector, do **image − object**.
• A negative first number means left; a negative second number means down.
• Two translations one after the other are the same as one translation by the **sum of the vectors**.
• The image is **congruent** to the object: same lengths, same angles, same way round.
• To undo a translation, **reverse the signs** of both parts.
• None of this needs a grid — the arithmetic on the coordinates is the whole method.

Key words: *translation · vector · object · image · congruent · corresponding points*

```
object (1, 1) (4, 1) (4, 3)
vector (−4, 1)
image (−3, 2) (0, 2) (0, 4)
(3, 2) right 3, up 2
(−5, 1) left 5, up 1
(4, −6) right 4, down 6
point (−1, 5), vector (3, −2)
```

14.4 Reflection

• A **reflection** flips a shape in a **mirror line**.
• Every point and its image are the **same distance** from the mirror, on opposite sides, measured **perpendicular** to it.
• Reflect in the **x-axis**: (x, y) → **(x, −y)**.
• Reflect in the **y-axis**: (x, y) → **(−x, y)**.
• Reflect in **y = x**: (x, y) → **(y, x)** — the coordinates swap.
• A point **on** the mirror line does not move. It is invariant.
• The image is **congruent**: every length and every angle is unchanged.
• But the shape is turned over, so the vertices run the other way round.
• To find an unknown mirror line, join a point to its image; the mirror cuts that join in half at right angles.

Key words: *reflection · mirror line · image · congruent · invariant point · perpendicular*

```
object at (3, 2)
mirror: the x-axis
image at (3, −2)
both 2 away from the line
(5, 3) → (5, −3)
(−1, 4) → (−1, −4)
(2, −6) → (2, 6)
```

14.5 Rotation

• A **rotation** turns a shape about a **centre of rotation**.
• To describe one fully you need **three** things: the angle, the direction and the centre.
• About the origin, **90° anticlockwise**: (x, y) → **(−y, x)**.
• About the origin, **180°**: (x, y) → **(−x, −y)**.
• About the origin, **90° clockwise**: (x, y) → **(y, −x)**.
• For 180° the direction makes no difference.
• The **centre does not move**. Everything else does.
• About any other centre, work with the offset from the centre, turn that, then add the centre back on.
• The image is **congruent**, and unlike a reflection the vertices still run the same way round.

Key words: *rotation · centre of rotation · anticlockwise · clockwise · quarter turn · half turn*

```
centre (0, 0)
angle 90°
direction anticlockwise
all three are needed
(3, 1) → (−1, 3)
(2, 4) → (−4, 2)
(−1, 3) → (−3, −1)
```

14.6 Enlargement

• An **enlargement** multiplies every length by a **scale factor**.
• It needs **two** things: the scale factor and the centre of enlargement.
• From the origin: (x, y) → **(kx, ky)**.
• From any other centre: multiply the **offset from the centre** by k, then add the centre back on.
• **Angles do not change.** Only lengths do.
• The image is **mathematically similar** to the object — same shape, different size. It is **not** congruent unless k = 1.
• **Lengths and perimeters** are multiplied by **k**.
• **Areas** are multiplied by **k^2**, because an area is a length times a length.
• The centre of enlargement is the one point that does not move.
• A scale drawing is an enlargement — section 14.2 was doing this all along.

Key words: *enlargement · scale factor · centre of enlargement · similar · congruent · ratio*

```
scale factor 2
3 cm → 6 cm
5 cm → 10 cm
angles unchanged
k = 2
(2, 3) → (4, 6)
(−1, 4) → (−2, 8)
```