

Name _____

Date _____

14.5 Rotation — worked answers

Rotate shapes 90° and 180° about a centre of rotation, identify a rotation together with its centre and angle, and recognise that the image is congruent to the object. · 64 questions

A · Fluency — Quarter and half turns about the origin.

- 1 Rotate the point (3, 1) **90° anticlockwise** about the origin.
Answer: (-1, 3)
A quarter turn anticlockwise about (0, 0) sends (x, y) to (-y, x): (3, 1) → (-1, 3).
- 2 Rotate the point (2, 4) **90° anticlockwise** about the origin.
Answer: (-4, 2)
A quarter turn anticlockwise about (0, 0) sends (x, y) to (-y, x): (2, 4) → (-4, 2).
- 3 Rotate the point (-1, 3) **90° anticlockwise** about the origin.
Answer: (-3, -1)
A quarter turn anticlockwise about (0, 0) sends (x, y) to (-y, x): (-1, 3) → (-3, -1).
- 4 Rotate the point (-4, 2) **90° anticlockwise** about the origin.
Answer: (-2, -4)
A quarter turn anticlockwise about (0, 0) sends (x, y) to (-y, x): (-4, 2) → (-2, -4).
- 5 Rotate the point (5, -2) **90° anticlockwise** about the origin.
Answer: (2, 5)
A quarter turn anticlockwise about (0, 0) sends (x, y) to (-y, x): (5, -2) → (2, 5).
- 6 Rotate the point (-3, -5) **90° anticlockwise** about the origin.
Answer: (5, -3)
A quarter turn anticlockwise about (0, 0) sends (x, y) to (-y, x): (-3, -5) → (5, -3).
- 7 Rotate the point (1, -6) **90° anticlockwise** about the origin.
Answer: (6, 1)
A quarter turn anticlockwise about (0, 0) sends (x, y) to (-y, x): (1, -6) → (6, 1).
- 8 Rotate the point (4, 4) **90° anticlockwise** about the origin.
Answer: (-4, 4)
A quarter turn anticlockwise about (0, 0) sends (x, y) to (-y, x): (4, 4) → (-4, 4).
- 9 Rotate the point (-2, -1) **90° anticlockwise** about the origin.
Answer: (1, -2)
A quarter turn anticlockwise about (0, 0) sends (x, y) to (-y, x): (-2, -1) → (1, -2).
- 10 Rotate the point (6, 3) **90° anticlockwise** about the origin.
Answer: (-3, 6)
A quarter turn anticlockwise about (0, 0) sends (x, y) to (-y, x): (6, 3) → (-3, 6).

- 11** Rotate the point $(-5, 1)$ **90° anticlockwise** about the origin.

Answer: $(-1, -5)$

A quarter turn anticlockwise about $(0, 0)$ sends (x, y) to $(-y, x)$: $(-5, 1) \rightarrow (-1, -5)$.

- 12** Rotate the point $(2, -3)$ **90° anticlockwise** about the origin.

Answer: $(3, 2)$

A quarter turn anticlockwise about $(0, 0)$ sends (x, y) to $(-y, x)$: $(2, -3) \rightarrow (3, 2)$.

- 13** Rotate the point $(3, 1)$ by **180°** about the origin.

Answer: $(-3, -1)$

A half turn about $(0, 0)$ sends (x, y) to $(-x, -y)$: $(3, 1) \rightarrow (-3, -1)$. Clockwise or anticlockwise makes no difference for 180°.

- 14** Rotate the point $(2, 4)$ by **180°** about the origin.

Answer: $(-2, -4)$

A half turn about $(0, 0)$ sends (x, y) to $(-x, -y)$: $(2, 4) \rightarrow (-2, -4)$. Clockwise or anticlockwise makes no difference for 180°.

- 15** Rotate the point $(-1, 3)$ by **180°** about the origin.

Answer: $(1, -3)$

A half turn about $(0, 0)$ sends (x, y) to $(-x, -y)$: $(-1, 3) \rightarrow (1, -3)$. Clockwise or anticlockwise makes no difference for 180°.

- 16** Rotate the point $(-4, 2)$ by **180°** about the origin.

Answer: $(4, -2)$

A half turn about $(0, 0)$ sends (x, y) to $(-x, -y)$: $(-4, 2) \rightarrow (4, -2)$. Clockwise or anticlockwise makes no difference for 180°.

- 17** Rotate the point $(5, -2)$ by **180°** about the origin.

Answer: $(-5, 2)$

A half turn about $(0, 0)$ sends (x, y) to $(-x, -y)$: $(5, -2) \rightarrow (-5, 2)$. Clockwise or anticlockwise makes no difference for 180°.

- 18** Rotate the point $(-3, -5)$ by **180°** about the origin.

Answer: $(3, 5)$

A half turn about $(0, 0)$ sends (x, y) to $(-x, -y)$: $(-3, -5) \rightarrow (3, 5)$. Clockwise or anticlockwise makes no difference for 180°.

- 19** Rotate the point $(1, -6)$ by **180°** about the origin.

Answer: $(-1, 6)$

A half turn about $(0, 0)$ sends (x, y) to $(-x, -y)$: $(1, -6) \rightarrow (-1, 6)$. Clockwise or anticlockwise makes no difference for 180°.

- 20** Rotate the point $(4, 4)$ by **180°** about the origin.

Answer: $(-4, -4)$

A half turn about $(0, 0)$ sends (x, y) to $(-x, -y)$: $(4, 4) \rightarrow (-4, -4)$. Clockwise or anticlockwise makes no difference for 180°.

- 21** Rotate the point $(-2, -1)$ by **180°** about the origin.

Answer: $(2, 1)$

A half turn about $(0, 0)$ sends (x, y) to $(-x, -y)$: $(-2, -1) \rightarrow (2, 1)$. Clockwise or anticlockwise makes no difference for 180°.

- 22** Rotate the point (6, 3) by **180°** about the origin.

Answer: (-6, -3)

A half turn about (0, 0) sends (x, y) to (-x, -y): (6, 3) → (-6, -3). Clockwise or anticlockwise makes no difference for 180°.

- 23** Rotate the point (-5, 1) by **180°** about the origin.

Answer: (5, -1)

A half turn about (0, 0) sends (x, y) to (-x, -y): (-5, 1) → (5, -1). Clockwise or anticlockwise makes no difference for 180°.

- 24** Rotate the point (2, -3) by **180°** about the origin.

Answer: (-2, 3)

A half turn about (0, 0) sends (x, y) to (-x, -y): (2, -3) → (-2, 3). Clockwise or anticlockwise makes no difference for 180°.

- 25** Rotate the point (3, 1) **90° clockwise** about the origin.

Answer: (1, -3)

90° clockwise is the same as 270° anticlockwise, which sends (x, y) to (y, -x): (3, 1) → (1, -3).

- 26** Rotate the point (2, 4) **90° clockwise** about the origin.

Answer: (4, -2)

90° clockwise is the same as 270° anticlockwise, which sends (x, y) to (y, -x): (2, 4) → (4, -2).

- 27** Rotate the point (-1, 3) **90° clockwise** about the origin.

Answer: (3, 1)

90° clockwise is the same as 270° anticlockwise, which sends (x, y) to (y, -x): (-1, 3) → (3, 1).

- 28** Rotate the point (-4, 2) **90° clockwise** about the origin.

Answer: (2, 4)

90° clockwise is the same as 270° anticlockwise, which sends (x, y) to (y, -x): (-4, 2) → (2, 4).

- 29** Rotate the point (5, -2) **90° clockwise** about the origin.

Answer: (-2, -5)

90° clockwise is the same as 270° anticlockwise, which sends (x, y) to (y, -x): (5, -2) → (-2, -5).

- 30** Rotate the point (-3, -5) **90° clockwise** about the origin.

Answer: (-5, 3)

90° clockwise is the same as 270° anticlockwise, which sends (x, y) to (y, -x): (-3, -5) → (-5, 3).

- 31** Rotate the point (1, -6) **90° clockwise** about the origin.

Answer: (-6, -1)

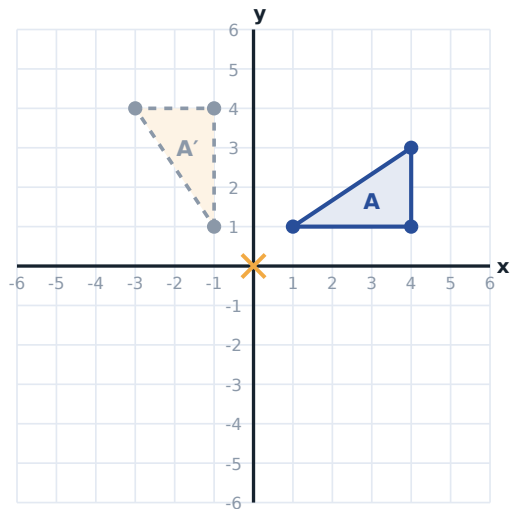
90° clockwise is the same as 270° anticlockwise, which sends (x, y) to (y, -x): (1, -6) → (-6, -1).

- 32** Rotate the point (4, 4) **90° clockwise** about the origin.

Answer: (4, -4)

90° clockwise is the same as 270° anticlockwise, which sends (x, y) to (y, -x): (4, 4) → (4, -4).

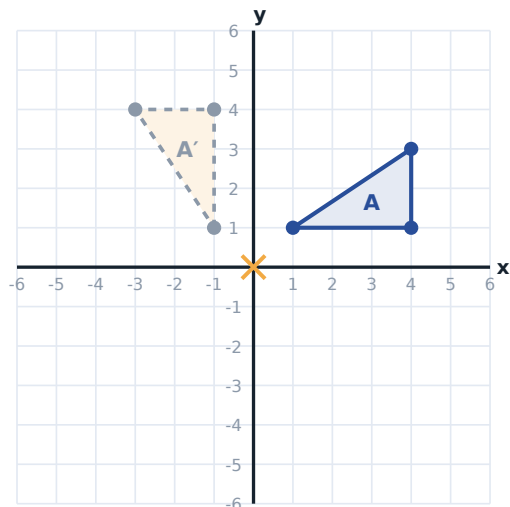
- 33** Triangle A has vertices $(1, 1)$, $(4, 1)$ and $(4, 3)$. It is rotated 90° anticlockwise about the origin. Where does $(1, 1)$ go?



Answer: $(-1, 1)$

Rotating 90° anticlockwise about $(0, 0)$ sends $(1, 1) \rightarrow (-1, 1)$. The centre of rotation is marked with a cross; it is the one point that does not move.

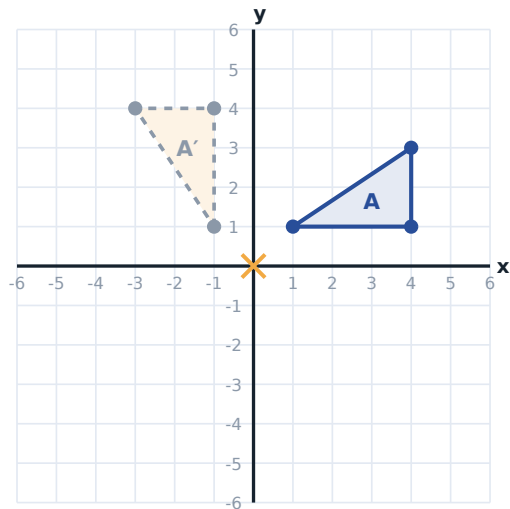
- 34** Triangle A has vertices $(1, 1)$, $(4, 1)$ and $(4, 3)$. It is rotated 90° anticlockwise about the origin. Where does $(4, 1)$ go?



Answer: $(-1, 4)$

Rotating 90° anticlockwise about $(0, 0)$ sends $(4, 1) \rightarrow (-1, 4)$. The centre of rotation is marked with a cross; it is the one point that does not move.

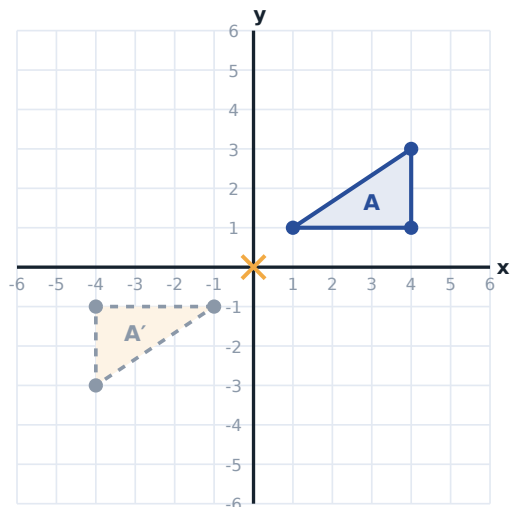
- 35** Triangle A has vertices $(1, 1)$, $(4, 1)$ and $(4, 3)$. It is rotated 90° anticlockwise about the origin. Where does $(4, 3)$ go?



Answer: $(-3, 4)$

Rotating 90° anticlockwise about $(0, 0)$ sends $(4, 3) \rightarrow (-3, 4)$. The centre of rotation is marked with a cross; it is the one point that does not move.

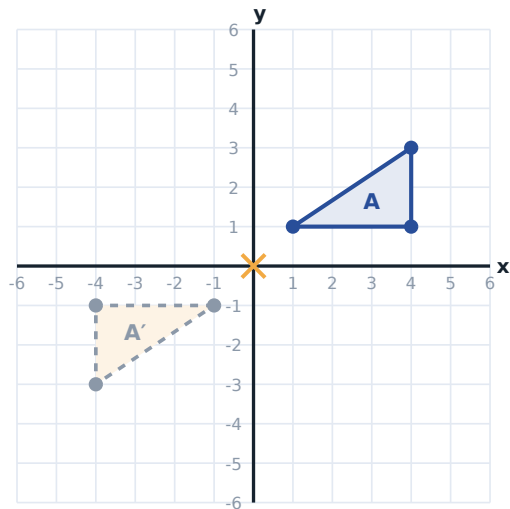
- 36** Triangle A has vertices $(1, 1)$, $(4, 1)$ and $(4, 3)$. It is rotated 180° about the origin. Where does $(1, 1)$ go?



Answer: $(-1, -1)$

Rotating 180° about $(0, 0)$ sends $(1, 1) \rightarrow (-1, -1)$. The centre of rotation is marked with a cross; it is the one point that does not move.

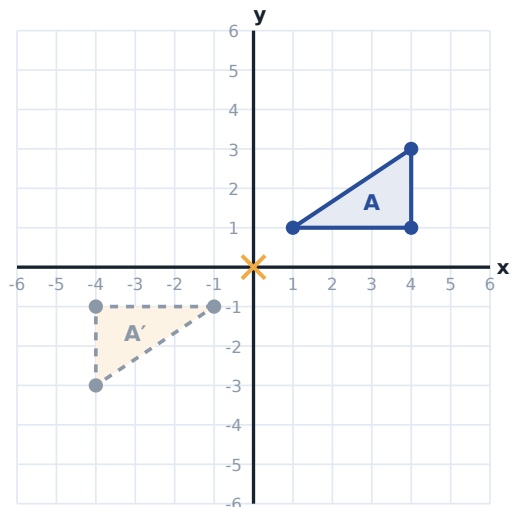
- 37** Triangle A has vertices $(1, 1)$, $(4, 1)$ and $(4, 3)$. It is rotated 180° about the origin. Where does $(4, 1)$ go?



Answer: $(-4, -1)$

Rotating 180° about $(0, 0)$ sends $(4, 1) \rightarrow (-4, -1)$. The centre of rotation is marked with a cross; it is the one point that does not move.

- 38** Triangle A has vertices $(1, 1)$, $(4, 1)$ and $(4, 3)$. It is rotated 180° about the origin. Where does $(4, 3)$ go?



Answer: $(-4, -3)$

Rotating 180° about $(0, 0)$ sends $(4, 3) \rightarrow (-4, -3)$. The centre of rotation is marked with a cross; it is the one point that does not move.

B · Reasoning — What a rotation needs, and what it preserves.

39 What does a rotation need before it can be carried out?

- ☐ A mirror line
- ☐ A centre, an angle and a direction
- ☐ A vector
- ☐ A scale factor

Answer: A centre, an angle and a direction

All three. Leaving out the centre is the commonest mistake.

40 After a rotation, the image is ...

- ☐ larger
- ☐ congruent to the object
- ☐ smaller
- ☐ similar but not congruent

Answer: congruent to the object

Turning a shape does not stretch it. Every length and angle is unchanged.

41 Why does the direction not matter for a rotation of 180° ?

- ☐ It always matters
- ☐ Half a turn each way ends in the same place
- ☐ 180° is special
- ☐ Because the centre is the origin

Answer: Half a turn each way ends in the same place

Going 180° clockwise and 180° anticlockwise both land you opposite where you started.

42 A rotation of 90° anticlockwise about the origin sends (x, y) to ...

- ☐ (y, x)
- ☐ $(-y, x)$
- ☐ $(y, -x)$
- ☐ $(-x, -y)$

Answer: $(-y, x)$

Check it on $(3, 0)$: a quarter turn anticlockwise takes it to $(0, 3)$, and the rule gives $(0, 3)$.

43 A rotation of 180° about the origin sends (x, y) to ...

- ☐ (y, x)
- ☐ $(-y, x)$
- ☐ $(-x, -y)$
- ☐ $(x, -y)$

Answer: $(-x, -y)$

Both signs change. It is the same as reflecting in both axes, one after the other.

44 Which point does not move during a rotation?

- ☐ The origin
- ☐ The centre of rotation
- ☐ Every point moves
- ☐ The first vertex

Answer: The centre of rotation

It is the pivot. It is only the origin when the origin is chosen as the centre.

45 Is 90° clockwise the same as 270° anticlockwise?

- ☐ Yes
- ☐ No

Answer: Yes

A quarter turn one way is three quarter turns the other.

46 Does a rotation turn a shape over?

- ☐ Yes
- ☐ No

Answer: No

The vertices still run the same way round. That is the difference between a rotation and a reflection.

47 Four rotations of 90° in the same direction leave a shape ...

- ☐ upside down
- ☐ exactly where it started
- ☐ reflected
- ☐ twice as big

Answer: exactly where it started

$4 \times 90 = 360$, a full turn.

48 How would you describe a transformation fully as a rotation?

- ☐ Just say "rotation"
- ☐ Give the angle, the direction and the centre
- ☐ Give the angle only
- ☐ Give the centre only

Answer: Give the angle, the direction and the centre

Marks in an examination are awarded for all three.

C · Challenge — Centres away from the origin, and combined turns.

49 Rotate the point (4, 2) 90° anticlockwise about the centre (1, 1).

Answer: (0, 4)

Work relative to the centre. (4, 2) is 3 across and 1 up from (1, 1). Turning 90° anticlockwise sends that offset to -1 across and 3 up, so the image is at (0, 4).

50 Rotate the point (5, 3) 180° about the centre (2, 1).

Answer: (-1, -1)

Work relative to the centre. (5, 3) is 3 across and 2 up from (2, 1). Turning 180° sends that offset to -3 across and -2 up, so the image is at $(-1, -1)$.

51 Rotate the point $(-1, 4)$ 90° anticlockwise about the centre (0, 2).

Answer: (-2, 1)

Work relative to the centre. $(-1, 4)$ is -1 across and 2 up from (0, 2). Turning 90° anticlockwise sends that offset to -2 across and -1 up, so the image is at $(-2, 1)$.

52 Rotate the point (3, -2) 180° about the centre (1, -1).

Answer: (-1, 0)

Work relative to the centre. (3, -2) is 2 across and -1 up from (1, -1). Turning 180° sends that offset to -2 across and 1 up, so the image is at $(-1, 0)$.

- 53** Rotate the point (6, 1) **90° clockwise** about the centre (3, 3).

Answer: (1, 0)

Work relative to the centre. (6, 1) is 3 across and -2 up from (3, 3). Turning 90° clockwise sends that offset to -2 across and -3 up, so the image is at (1, 0).

- 54** Rotate the point (-2, -3) **90° anticlockwise** about the centre (-1, -1).

Answer: (1, -2)

Work relative to the centre. (-2, -3) is -1 across and -2 up from (-1, -1). Turning 90° anticlockwise sends that offset to 2 across and -1 up, so the image is at (1, -2).

- 55** A shape of area 20 is rotated 90° about (3, 4). What is the area of the image?

Answer: 20

Rotation is congruent, so the area is unchanged at 20.

- 56** A point is rotated 90° anticlockwise about the origin, then 90° anticlockwise again. Name the single transformation that does the same job.

Answer: a rotation of 180° about the origin

$90 + 90 = 180$, about the same centre.

D · Word problems — Clocks, robots, tiles and fairground rides.

- 57** A minute hand tip is at (0, 5) on a clock face centred on the origin. It turns 90° clockwise. Where is the tip now?

Answer: (5, 0)

90° clockwise sends (x, y) to (y, -x): (0, 5) → (5, 0). On a clock that is a quarter of an hour.

- 58** A robot arm has its tool at (3, 2) and pivots about the origin. It rotates 180°. Where is the tool?

Answer: (-3, -2)

Both signs change: (3, 2) → (-3, -2).

- 59** A tile has a corner at (2, 5). The tile is turned a quarter turn anticlockwise about the origin. Where is that corner?

Answer: (-5, 2)

(x, y) → (-y, x), so (2, 5) → (-5, 2).

- 60** A fairground ride seat is at (6, 0). The ride turns 180° about its centre at (2, 0). Where is the seat?

Answer: (-2, 0)

The seat is $6 - 2 = 4$ to the right of the centre, so after half a turn it is 4 to the left: $2 - 4 = -2$, giving (-2, 0).

- 61** A logo is rotated four times by 90° about its centre. How many degrees has it turned altogether?

Answer: 360

$4 \times 90 = 360$ — a full turn, back to the start.

- 62** A pattern turns the point (1, 3) by 90° anticlockwise about (1, 1). Where does it land?

Answer: (-1, 1)

The point is 0 across and $3 - 1 = 2$ up from the centre. A quarter turn anticlockwise sends that offset to 2 left and 0 up, so the image is at (-1, 1).

- 63** A square window frame is rotated 180° . How many of its sides have changed length?

Answer: 0

None. A rotation is congruent, so every length is exactly as it was.

- 64** A shape at $(4, 1)$ has an image at $(-4, -1)$ with the origin as centre. What angle was it turned through?

Answer: 180°

Both coordinates have changed sign, which is a half turn.