Name ____________________ Date ____________

Unit 13 · Probability — teaching slides

18 slides, formatted for printing.

13.1 The probability scale

The scale from 0 to 1

- Every probability sits somewhere between 0 and 1.
- **0** means it can never happen.
- **1** means it must happen.
- Nothing can sit outside that range — not −0.2, not 1.5.

```
0 impossible
1/4 unlikely
1/2 even chance
3/4 likely
1 certain
```

Three ways to write it

- A probability can be a fraction, a decimal or a percentage.
- All three say the same thing.
- Choose whichever suits the question.
- But convert to one form before comparing two probabilities.

```
1/5 = 0.2 = 20%
1/2 = 0.5 = 50%
3/4 = 0.75 = 75%
```

Why words are not enough

- “Probably” means different things to different people.
- One person's “likely” is another's “even chance”.
- A number on the scale is exact and shared.
- That is the whole reason probability is written as a number.

```
“it will probably rain” → how likely?
“a 70% chance of rain” → exactly this likely
```

Outcomes and events

- An **outcome** is one possible result of a trial.
- An **event** is the outcome or outcomes you are interested in.
- Rolling a 4 is an outcome; rolling an even number is an event.
- Getting the distinction right makes the counting easier later.

```
dice outcomes: 1, 2, 3, 4, 5, 6
event “even”: 2, 4, 6
three outcomes make up that one event
```

An event and its opposite

- Either something happens or it does not.
- So the two probabilities must add to 1.
- If P(rain) = 0.7 then P(no rain) = 0.3.
- This is the quickest calculation in the whole unit — use it.

```
P(win) = 0.35
P(not win) = 1 − 0.35 = 0.65
```

What a probability does not promise

- A probability of 1/2 does not mean exactly half of ten tosses are heads.
- It describes the long run, not any particular set of trials.
- Six heads in ten tosses is entirely ordinary.
- This is the idea that section 13.3 returns to.

```
10 tosses of a fair coin
6 heads, 4 tails – perfectly normal
the coin is not unfair
```

13.2 Calculating probabilities

The formula

- Count the outcomes that make the event happen.
- Count all the possible outcomes.
- Divide the first by the second.
- It only works when every outcome is equally likely.

```
dice, event “even”
favourable: 2, 4, 6 → 3
total: 1 to 6 → 6
P = 3/6 = 1/2
```

Count the total carefully

- The total is the denominator of every answer.
- One miscount and every part of the question is wrong.
- Add up a bag's contents before doing anything else.
- Write the total down where you can see it.

```
4 red + 5 blue + 3 green
total = 12
every answer will be something over 12
```

Everything adds to 1

- One of the possible outcomes must happen.
- So all their probabilities total exactly 1.
- That is a useful check on a list of probabilities.
- And it gives you the complement rule.

```
P(red) = 4/12
P(blue) = 5/12
P(green) = 3/12
total = 12/12 = 1 ✓
```

The complement

- Either an event happens or it does not.
- So **P(not A) = 1 − P(A)**.
- Often far quicker than counting the other outcomes.
- Especially when “not A” covers many possibilities.

```
P(blue) = 5/12
P(not blue) = 1 − 5/12 = 7/12
quicker than counting 4 + 3
```

Mutually exclusive events

- Two events that cannot both happen at once.
- Rolling a 2 and rolling a 5 on the same roll.
- For these, the probabilities simply add.
- Careful: “even” and “prime” are *not* exclusive — 2 is both.

```
P(red or green)
= 4/12 + 3/12
= 7/12
```

When the formula does not apply

- It needs every outcome to be **equally likely**.
- A fair dice qualifies; a bent one does not.
- A drawing pin is more likely to land one way than the other.
- In those cases you have to experiment — which is 13.3.

```
fair dice → use the formula
drawing pin → experiment instead
weighted spinner → experiment instead
```

13.3 Experiments and expected results

Two kinds of probability

- **Theoretical**: worked out from the outcomes, before any trial.
- **Experimental**: measured from trials that actually happened.
- Both are written on the same 0-to-1 scale.
- They will rarely be exactly equal, and that is not a problem.

```
fair dice, P(six) theoretical = 1/6
60 rolls gave 8 sixes
experimental = 8/60 = 2/15
```

Expected number

- Multiply the probability by the number of trials.
- That is how many times you would expect the event.
- It is an average over many repeats.
- Not a prediction of exactly what will happen this time.

```
P = 1/6, trials = 60
expected = 1/6 × 60 = 10
actual might be 8, or 13
```

Variation is normal

- Short runs vary a great deal.
- 7 heads in 10 tosses does not make a coin unfair.
- Expect the experimental value to wander around the theoretical one.
- Judge fairness on hundreds of trials, not ten.

```
10 tosses: 7 heads → 70%
100 tosses: 54 heads → 54%
1000 tosses: 508 heads → 50.8%
```

More trials, closer agreement

- As the number of trials grows, the experimental value settles.
- It moves towards the theoretical probability.
- It never becomes exact — but it becomes trustworthy.
- This is the same idea as sample size in Unit 6.

```
the more you roll,
the closer the proportion of sixes
gets to 1/6
```

When theory cannot help

- The formula needs every outcome to be equally likely.
- A drawing pin, a bottle top, a weighted spinner — none qualify.
- For those, run an experiment and count.
- The experimental probability is then the only one you have.

```
drawing pin, 200 drops
point-up 132 times
P(point-up) ≈ 132/200 = 0.66
```

Spotting bias

- Compare what happened with what was expected.
- A small difference means nothing.
- A large difference over many trials suggests bias.
- Say how many trials you used — without it the claim means nothing.

```
fair 4-sector spinner, 200 spins
expected red: 50
actual red: 90
that is a lot — worth investigating
```