nexamind EDUCATION

Name _____ Date _____

Unit 12 · Ratio and proportion — teaching slides

18 slides, formatted for printing.

12.1 Understanding ratio

What a ratio says

- A **ratio** compares two amounts, part with part.
- 3 : 2 means three of the first for every two of the second.
- It says nothing about the actual sizes — only the comparison.
- 3 : 2 could be 3 and 2, or 30 and 20, or 300 and 200.

```
squash 1 : 4
1 part syrup to 4 parts water
so 50 ml syrup needs 200 ml water
```

Order matters

- The first number belongs to the first thing named.
- 3 : 2 and 2 : 3 describe completely different mixtures.
- Always write the ratio in the order the question names them.
- Label it if there is any doubt: boys : girls = 2 : 3.

```
boys : girls = 2 : 3
girls : boys = 3 : 2
same class, different ratio
```

Simplifying

- Divide every part by their highest common factor.
- The comparison is unchanged; only the numbers get smaller.
- This is exactly what you do to a fraction.
- With three parts, use the HCF of all three.

```
12 : 18
HCF is 6
= 2 : 3
45 : 60 : 75 → 3 : 4 : 5
```

Equivalent ratios

- Multiply every part by the same number.
- Or divide every part by the same number.
- Whatever you do to one part, do to them all.
- This is how you scale a recipe up or down.

```
2 : 3
× 4 → 8 : 12
× 10 → 20 : 30
```

Same units first

- A ratio of measurements needs both in the same unit.
- Convert before you compare — never after.
- Then the units cancel and the ratio is just numbers.
- A ratio has no units of its own.

```
40 cm : 1 m
= 40 cm : 100 cm
= 2 : 5
```

Ratio and fraction are different

- A ratio compares part with **part**.
- A fraction compares part with **whole**.
- In 2 : 3 there are 5 parts altogether.
- So the first share is 2/5 of the whole — not 2/3.

```
ratio 2 : 3
total parts 2 + 3 = 5
first share = 2/5
second share = 3/5
```

12.2 Sharing in a ratio

Three steps, always the same

- **Add** the ratio numbers to find the total parts.
- **Divide** the quantity by that to find one part.
- **Multiply** each ratio number by one part.
- Add, divide, multiply — in that order, every time.

```
share 20 in the ratio 2 : 3
parts: 2 + 3 = 5
one part: 20 ÷ 5 = 4
shares: 8 and 12
```

Always check

- Add your two shares together.
- They must give back the original total.
- If they do not, something went wrong.
- This takes five seconds and catches nearly every error.

```
8 + 12 = 20 ✓
if you had 8 and 10, the check fails
```

Three-part ratios

- Exactly the same method — nothing new to learn.
- Add all three, divide, then multiply each.
- The check still works: all three shares must total the quantity.
- Keep the shares in the order the ratio names them.

```
share 60 in 1 : 2 : 3
parts: 6, one part: 10
shares: 10, 20, 30
check: 10 + 20 + 30 = 60 ✓
```

Working backwards from one share

- Sometimes you are told one share, not the total.
- Divide that share by its ratio number to find one part.
- Then multiply by the total parts to reach the whole.
- The value of one part is the key to everything here.

```
ratio 3 : 5, smaller share is 27
one part: 27 ÷ 3 = 9
total: 8 × 9 = 72
```

When the shares differ by a given amount

- The difference between the shares is also made of parts.
- Subtract the ratio numbers to find how many parts the difference is.
- Divide the given difference by that.
- Then carry on as usual.

```
ratio 5 : 3, shares differ by 16
difference: 5 − 3 = 2 parts
one part: 16 ÷ 2 = 8
shares: 40 and 24
```

When it does not divide exactly

- A total can only be shared exactly if it divides by the total parts.
- 10 shared 2 : 3 : 4 needs 9 parts — and 10 is not a multiple of 9.
- In a real problem, say so rather than forcing an answer.
- Checking divisibility first saves wasted work.

```
20 in 2 : 3 → 5 parts, 20 ÷ 5 = 4 ✓
10 in 2 : 3 : 4 → 9 parts, 10 ÷ 9 ✗
```

12.3 Direct proportion

What direct proportion means

- Double one quantity and the other doubles too.
- Halve one and the other halves.
- Dividing one by the other always gives the same number.
- That constant is the **rate**.

```
3 pens cost 12 → 4 each
6 pens cost 24 → 4 each
9 pens cost 36 → 4 each
```

The unitary method

- Find the value of **one** first.
- Then multiply to get however many you need.
- Divide, then multiply — two steps, always in that order.
- It works for almost every proportion question you will meet.

```
5 pens cost 35
one pen: 35 ÷ 5 = 7
8 pens: 7 × 8 = 56
```

Reading it from a table

- Divide each pair and see whether you always get the same number.
- If you do, the quantities are directly proportional.
- If you do not, they are not — however neat the table looks.
- That constant is the rate you then use.

```
litres: 2 4 6
cost: 10 20 30
10 ÷ 2 = 20 ÷ 4 = 30 ÷ 6 = 5 ✓
```

Reading it from a graph

- Direct proportion gives a straight line.
- And it passes through the **origin**.
- Zero of one really does give zero of the other.
- A line that misses the origin is not direct proportion.

```
petrol cost → through (0, 0) ✓
taxi fare → starts at (0, 5) ✗
```

The zero test

- Ask: does zero of one give zero of the other?
- Zero litres of petrol costs nothing — direct.
- Zero kilometres in a taxi still costs the fixed charge — not direct.
- One question settles it, without any calculation.

```
0 apples cost 0 → direct
0 km costs $5 → not direct
```

Inverse proportion

- Sometimes one goes **up** as the other goes **down**.
- Twice as many workers, half the time.
- Here the **product** stays constant, not the ratio.
- Read the situation before choosing a method.

```
3 workers × 6 hours = 18
6 workers × 3 hours = 18
the product is fixed
```