nexamind EDUCATION

Name Date

Unit 1 · Integers — teaching slides

36 slides, formatted for printing.

1.1 Adding and subtracting integers

What is an integer?

- Integers are whole numbers — they have no fractional part.
- They can be positive, negative, or zero.
- Numbers like 2.5, ¾ and √2 are *not* integers.

```
Integers: −200, −7, 0, 1, 43
Not integers: 0.5, −2¼, √10
```

The number line

- Every integer has a position on the number line.
- Moving **right** makes a number bigger; moving **left** makes it smaller.
- So −9 is smaller than −2, even though 9 is bigger than 2.

```
−5 −4 −3 −2 −1 0 1 2 3 4 5
−9 < −2 < 0 < 6
```

Adding integers

- To add a **positive** integer, move right.
- To add a **negative** integer, move left — it has the same effect as subtracting.
- Rule: a + (−b) = a − b

```
−8 + 5 = −3 (start at −8, move 5 right)
6 + (−9) = 6 − 9 = −3
```

Subtracting integers

- To subtract a **positive** integer, move left.
- To subtract a **negative** integer, move right — the two minus signs cancel.
- Rule: a − (−b) = a + b

```
−7 − 4 = −11
−12 − (−5) = −12 + 5 = −7
4 − (−6) = 4 + 6 = 10
```

Inverse operations

- Addition and subtraction undo each other.
- Use this to find a missing number in a calculation.
- If a + b = c then b = c − a.

```
−9 + ? = 4 → ? = 4 − (−9) = 13
? − (−6) = −2 → ? = −2 + (−6) = −8
```

Where you meet this in real life

- Temperatures below zero.
- Depths below sea level.
- Money you owe (an overdrawn bank balance).
- Floors below ground in a building; scores under par in golf.

```
−7 °C rises by 12 °C → 5 °C
A submarine at −85 m descends 40 m → −125 m
```

1.2 Multiplying and dividing integers

Multiplication is repeated addition

- 3 × 4 means 4 + 4 + 4 = 12.
- In the same way, 3 × (−4) means (−4) + (−4) + (−4) = −12.
- So a positive times a negative is negative.

```
5 × (−2) = (−2)+(−2)+(−2)+(−2)+(−2) = −10
```

The sign rules

- + × + = +
- + × − = −
- − × + = −
- − × − = +
- Short version: **same signs → positive, different signs → negative**.

```
(−8) × (−5) = 40
9 × (−4) = −36
```

Division follows the same rules

- Division is the inverse of multiplication, so the signs behave identically.
- Because −3 × 4 = −12, it follows that −12 ÷ 4 = −3 and −12 ÷ (−3) = 4.

```
−48 ÷ 6 = −8
−72 ÷ (−9) = 8
56 ÷ (−7) = −8
```

Chains of negatives

- Multiply step by step, left to right.
- Or count the negative signs first:
- even number of negatives → positive answer;
- odd number of negatives → negative answer.

```
−3 × (−4) × (−2) = −24 (3 negatives → negative)
−2 × (−3) × (−4) × (−5) = 120 (4 negatives → positive)
```

Squaring and cubing negatives

- $(-5)^2 = (-5) \times (-5) = 25$ — always positive.
- $(-2)^3 = (-2) \times (-2) \times (-2) = -8$ — stays negative.
- Squaring any integer (positive or negative) gives a positive result.

```
(−10)² = 100
(−3)³ = −27
```

Order of operations still applies

- Brackets → Indices → Division and Multiplication → Addition and Subtraction.
- Do the multiplying and dividing before the adding and subtracting.

```
−3 × 4 + (−5) × 2 = −12 + (−10) = −22
20 ÷ (−4) − 3 × (−6) = −5 + 18 = 13
```

1.3 Lowest common multiples

Multiples

- A multiple of n is $n \times 1$, $n \times 2$, $n \times 3$, …
- Multiples go on forever.
- Every number is a multiple of itself and of 1.

```
Multiples of 4: 4, 8, 12, 16, 20, 24, 28, …
Multiples of 6: 6, 12, 18, 24, 30, 36, …
```

Common multiples

- A common multiple appears in *both* lists.
- There are infinitely many common multiples of any two numbers.

```
Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36 …
Multiples of 6: 6, 12, 18, 24, 30, 36 …
Common multiples of 4 and 6: 12, 24, 36, …
```

The lowest common multiple

- The LCM is the smallest common multiple.
- For 4 and 6 the common multiples are 12, 24, 36, … so the LCM is 12.

```
LCM(4, 6) = 12
LCM(9, 12) = 36
LCM(6, 7) = 42
```

An efficient method

- Listing every multiple of both numbers is slow.
- Instead: **list multiples of the larger number only**, and test each one to see if the smaller number divides into it.
- The first one that works is the LCM.

```
LCM of 6 and 10 → multiples of 10: 10 ✗, 20 ✗, 30 ✓ (30 = 6 × 5)
So LCM(6, 10) = 30
```

Two useful shortcuts

- If the numbers share no common factor, multiply them.
- If one is a multiple of the other, the LCM is the larger one.

```
LCM(5, 8) = 40 (no shared factors)
LCM(4, 12) = 12 (12 is already a multiple of 4)
```

Why LCM is useful

- Events that repeat at different intervals meet again after the LCM.
- Buses, flashing lights, running laps, packing items into equal groups.

```
Buses every 12 min and 18 min → together again after LCM = 36 min
Lights every 20 s and 25 s → together again after 100 s
```

1.4 Highest common factors

Factors

- A factor of n divides into n exactly, leaving no remainder.
- Find them in pairs so you do not miss any.
- Every number has 1 and itself as factors.

```
Factors of 24: 1 × 24, 2 × 12, 3 × 8, 4 × 6 → 1, 2, 3, 4, 6, 8, 12, 24
```

Common factors

- A common factor divides into both numbers.
- Every pair of numbers has at least one common factor: 1.

```
Factors of 12: 1, 2, 3, 4, 6, 12
Factors of 18: 1, 2, 3, 6, 9, 18
Common factors: 1, 2, 3, 6
```

The highest common factor

- The HCF is the largest common factor.
- For 12 and 18 the common factors are 1, 2, 3, 6 — so the HCF is 6.

```
HCF(12, 18) = 6
HCF(24, 36) = 12
HCF(9, 20) = 1
```

An efficient method

- List the factors of the **smaller** number.
- Start from the largest and work down.
- The first one that also divides the larger number is the HCF.

```
HCF of 16 and 24 → factors of 16: 16 ✗ (24 ÷ 16 is not whole), 8 ✓ (24 ÷ 8 = 3)
So HCF(16, 24) = 8
```

Co-prime numbers

- If the only common factor is 1, the numbers are **co-prime**.
- A fraction whose numerator and denominator are co-prime cannot be simplified.

```
HCF(9, 20) = 1 → 9 and 20 are co-prime
25/36 cannot be simplified because HCF(25, 36) = 1
```

Using the HCF

- Simplify a fraction in one step by dividing by the HCF.
- Split things into the largest possible equal groups.
- Remember HCF × LCM = product of the two numbers.

```
24/36: HCF = 12 → 2/3
48 red and 36 blue pens → HCF = 12 identical packs
```

1.5 Tests for divisibility

What does divisible mean?

- A number is divisible by d when the division leaves no remainder.
- The answer must be a whole number (an integer).
- Divisibility tests let you check without doing the division.

```
15 ÷ 5 = 3, so 15 is divisible by 5
15 ÷ 4 = 3 remainder 3, so 15 is not divisible by 4
```

The easy tests: 2, 5 and 10

- ÷ 2 — the last digit is even (0, 2, 4, 6 or 8).
- ÷ 5 — the last digit is 0 or 5.
- ÷ 10 — the last digit is 0.

```
4 530 → last digit 0 → divisible by 2, 5 and 10
2 915 → last digit 5 → divisible by 5, not by 2 or 10
```

Digit-sum tests: 3 and 9

- ÷ 3 — the sum of the digits is divisible by 3.
- ÷ 9 — the sum of the digits is divisible by 9.
- You can add the digits again if the total is still large.

```
342 → 3 + 4 + 2 = 9 → divisible by 3 and by 9
8 514 → 8 + 5 + 1 + 4 = 18 → divisible by 3 and by 9
2 461 → 2 + 4 + 6 + 1 = 13 → divisible by neither
```

Last-digits tests: 4 and 8

- ÷ 4 — the number formed by the last two digits is divisible by 4.
- ÷ 8 — the number formed by the last three digits is divisible by 8.
- This works because 100 is a multiple of 4 and 1000 is a multiple of 8.

```
5 060 → last two digits 60, and 60 ÷ 4 = 15 → divisible by 4
7 128 → last three digits 128, and 128 ÷ 8 = 16 → divisible by 8
1 234 → last two digits 34 → not divisible by 4
```

Combining tests: 6, 12 and beyond

- ÷ 6 — it passes the tests for both 2 and 3.
- ÷ 12 — passes the tests for 3 and 4.
- This shortcut only works when the two divisors are co-prime.
- 2 and 4 share a factor, so passing both does *not* prove divisibility by 8.

```
3 456 → even ✓ and digits add to 18 ✓ → divisible by 6
12 is divisible by 2 and 4 but 12 ÷ 8 = 1.5 ✗
```

The test for 11

- Add the digits in the odd positions (counting from the right).
- Add the digits in the even positions.
- If the difference is 0 or a multiple of 11, the number is divisible by 11.

```
5 148 → (8 + 1) − (4 + 5) = 0 → divisible by 11
5 148 ÷ 11 = 468 ✓
```

1.6 Square roots and cube roots

Square numbers

- Multiply an integer by itself and you get a square number.
- Write it with an index of 2: 7^2 means 7×7.
- You read 7^2 as “seven squared”.
- They are called square numbers because that many dots make a square.

```
1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225
12² = 144
```

Square roots

- The square root is the inverse of squaring.
- $\sqrt{144} = 12$ because $12 \times 12 = 144$.
- The symbol √ is called a radical sign.
- Learn the square numbers up to 15^2 and you can read square roots straight off.

```
√81 = 9
√400 = 20
√121 = 11
```

Cube numbers

- Multiply an integer by itself, then by itself again.
- Write it with an index of 3: 4^3 means $4 \times 4 \times 4$.
- You read 4^3 as “four cubed”.
- They are called cube numbers because that many small cubes build a larger cube.

```
1, 8, 27, 64, 125, 216, 343, 512, 729, 1000
4³ = 64
```

Cube roots

- The cube root is the inverse of cubing.
- $\sqrt[3]{27} = 3$ because $3 \times 3 \times 3 = 27$.
- The symbol has a small 3 in it: $\sqrt[3]{}$
- Unlike square roots, cube roots of negative numbers exist: $\sqrt[3]{(-8)} = -2$.

```
∛125 = 5
∛1000 = 10
∛216 = 6
```

Estimating square roots

- Most numbers are not square numbers, so their square root is not a whole number.
- Trap the number between the two nearest square numbers.
- Then decide which end it is closer to.

```
49 < 50 < 64 → 7 < √50 < 8
64 < 79 < 81 → √79 is between 8 and 9, and much closer to 9
196 < 215 < 225 → √215 is between 14 and 15
```

Putting it together

- Roots follow the normal order of operations — treat them like brackets.
- Work out what is under the root sign first.

```
√121 + ∛64 = 11 + 4 = 15
√(9 × 16) = √144 = 12
(−3)³ + √64 = −27 + 8 = −19
```